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STABILITY OF CERTAIN NEUTRAL TYPE DIFFERENTIAL EQUATION AND NUMERICAL EXPERIMENT VIA DIFFERENTIAL TRANSFORM METHOD

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Abstract. In this study, we obtain both the asymptotically stability and the numerical solution of first order neutral type differential equation with multiple retarded arguments. We first obtain sufficient specific conditions expressed in terms of linear matrix inequality (LMI) using the Lyapunov method to establish the asymptotic stability of solutions. Secondly, we use the differential transform method (DTM) to show numerical solutions. Finally, two examples are presented to demonstrate the effectiveness and applicability of proposed methods by Matlab and an appropriate computer program.

Keywords: Stability, Lyapunov method, LMI, DTM.

2010 Mathematics Subject Classification: 34K20, 34K40, 65L10.

1. Introduction

The different particular cases of delay differential equations have been searched by many researchers for the past few decades. Recently, it can be seen from the related literature that qualitative properties of various neutral differential equations have been investigated by many authors and the researchers have obtained many interesting and important results on some qualitative properties such as stability, exponentially stability, asymptotically stability, oscillation, non-oscillations of solutions and etc.(see,[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]).

DTM, which is a semi-analytical-numerical technique, is based on the Taylor series expansion. The concept of method was first introduced by Pukhov [15] to solve linear and nonlinear problems in physical processes, and by Zhou [16] to study electrical circuits. This

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method is advantageous in obtaining numerical, analytical and exact solutions of ordinary and partial differential equations it has been widely studied and applied in recent years (see,[17, 18, 19, 20, 21, 22, 23, 24, 25]). According to the current techniques in the literature, DTM is a reliable method that requires less work and does not require linearization.

In this study, we consider the following first order neutral type differential equation with multiple retarded arguments:

$$\frac{d}{dt}[x(t)+p(t)x(t-\tau)]+a(t)f(x(t))+b(t)g(x(t-\sigma))+c(t)\int_{t-\delta}^{t}x(s)ds=0, \qquad (1.1)$$

where $p(t), a(t),\ b(t), c(t) : [t_0, \infty) \to [0, \infty),\ t_0 \geq 0,$ and $f,\ g : \Re \to \Re$ with $f(0) = 0, g(0) = 0$ are continuous functions on their respective domains;τ, σ and δ are positive real constants. For each solution $x(t)$ of equation 1.1, we assume the existence following initial condition:

$$x(\theta) = \Phi(\theta), \quad \theta \in [t_0 - H, t_0],$$

where $\Phi \in C([t_0 - H, t_0], R), H = \max\{\tau, \sigma, \delta\}$.
Define

$$h_1(x) = \begin{cases} \frac{h(x)}{x}, & x \neq 0 \\ \\ \frac{dh(0)}{dt}, & x = 0 \end{cases} \qquad (1.2)$$

and

$$g_1(x) = \begin{cases} \frac{g(x)}{x}, x \neq 0 \\ \\ \frac{dg(0)}{dt}, x = 0. \end{cases} \qquad (1.3)$$

The main purpose and contribution of this work can be summarized as follows aspects:

i. This research on the stability of certain neutral type differential equation and their numerical solutions is still at the stage of developing. Therefore, we propose a novel stability criterion for further improvements.

ii. The proof technique for the asymptotically stability of the equation considered in this study includes the Lyapunov function method and the LMI technique. Also, DTM is used to obtain numerical solutions of the equation considered.

iii. The simulations showing the behaviors of the solutions of the equation addressed by applying the Lyapunov method and the numerical solutions of the equation addressed using DTM show that the proposed methods are useful and efficient.

2. Preliminaries and stability results

We suppose that there exist nonnegative constants $a_i, b_i, c_i, m_i, n_i \quad (i = 1, 2)$ and p_1 such that for $t \geq 0$,

$$a_1 \leq a(t) \leq a_2,\ b_1 \leq b(t) \leq b_2,\ c_1 \leq c(t) \leq c_2, \tag{2.4}$$

$$|p(t)| \leq p_1 < 1,\ m_1 \leq f_1(x) \leq m_2,\ n_1 \leq g_1(x) \leq n_2. \tag{2.5}$$

For convenience, define the operator $D : \Re \to \Re$ as

$$D(x_t) = x(t) + p(t)x(t-\tau) - \alpha \int_{t-\tau}^{t} x(s)ds - \beta \int_{t-\sigma}^{t} x(s)ds,$$

where α ve β are positive scalars to be chosen later. From 1.2 and 1.3, equation 1.1 can be readily rewritten as follows for $t \geq 0$,

$$\frac{d}{dt}[x(t) + p(t)x(t-\tau) - \alpha \int_{t-\tau}^{t} x(s)ds - \beta \int_{t-\sigma}^{t} x(s)ds] = -(f_1(x)a(t) + \alpha + \beta)x(t)$$
$$+\alpha x(t-\tau) + \beta x(t-\sigma) - g_1(x(t-\sigma))b(t)x(t-\sigma) - c(t) \int_{t-\delta}^{t} x(s)ds. \tag{2.6}$$

Theorem 2.1. *Let $a_i,\ b_i,\ c_i,\ m_i$ and n_i $(i = 1,\ 2)$ be nonnegative constants. Then trivial solution of neutral type differential equation 2.6 is asymptotically stability if the operator D is stable and there exist positive constants $\tau, \sigma, \delta, \alpha,\ \beta$ and λ_j $(j = 1,\ 2, \ldots, 5)$ such that*

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \beta - n_1 b_1 & \Pi_{14} & \Pi_{15} & -c_1 \\ * & \Pi_{22} & \Pi_{23} & -\alpha^2 & -\alpha\beta & -p_1 c_1 \\ * & * & -\lambda_2 & \Pi_{34} & \Pi_{35} & 0 \\ * & * & * & -\lambda_3 & 0 & \alpha c_2 \\ * & * & * & * & -\lambda_4 & \beta c_2 \\ * & * & * & * & * & -\lambda_5 \end{bmatrix} < 0, \tag{2.7}$$

where $\Pi_{11} = -2(m_1 a_1 + \alpha + \beta) + \lambda_1 + \lambda_2 + \lambda_3 \tau^2 + \lambda_4 \sigma^2 + \lambda_5 \delta^2, \Pi_{12} = \alpha - (m_1 a_1 + \alpha + \beta)p_1,$ $\Pi_{14} = m_2 a_2 \alpha + \alpha^2 + \alpha\beta, \Pi_{15} = m_2 a_2 \beta + \alpha\beta + \beta^2, \Pi_{22} = 2\alpha p_1 - \lambda_1, \Pi_{23} = \beta p_1 - n_1 b_1 p_1,$ $\Pi_{34} = -\alpha\beta + \alpha n_2 b_2, \Pi_{35} = -\beta^2 + \beta n_2 b_2$ *and the symbols "$*$" shows the elements below the main diagonal of the symmetric matrix* Π .

Proof. Consider the appropriate Lyapunov functional as

$$V(t) =[D(x_t)]^2 + \lambda_1 \int_{t-\tau}^{t} x^2(s)ds + \lambda_2 \int_{t-\sigma}^{t} x^2(s)ds + \lambda_3\tau \int_{t-\tau}^{t} (\tau - t + s)x^2(s)ds$$
$$+ \lambda_4\sigma \int_{t-\sigma}^{t} (\sigma - t + s)x^2(s)ds + \lambda_5\delta \int_{t-\delta}^{t} (\delta - t + s)x^2(s)ds,$$

where $D(x_t) = x(t) + p(t)x(t-\tau) - \alpha \int\limits_{t-\tau}^{t} x(s)ds - \beta \int\limits_{t-\sigma}^{t} x(s)ds.$

When the time derivative of $V(t)$ along the trajectory of equation 2.6 are calculate, we obtain

$$\frac{dV}{dt} =2[x(t) + p(t)x(t-\tau) - \alpha \int_{t-\tau}^{t} x(s)ds - \beta \int_{t-\sigma}^{t} x(s)ds]$$
$$\times [-(f_1(x)a(t) + \alpha + \beta)x(t) + \alpha x(t-\tau) + \beta x(t-\sigma)$$
$$- g_1(x(t-\sigma))b(t)x(t-\sigma) - c(t) \int_{t-\delta}^{t} x(s)ds] + \lambda_1[x^2(t) - x^2(t-\tau)]$$
$$+ \lambda_2[x^2(t) - x^2(t-\sigma)] + \lambda_3\tau^2x^2(t) - \lambda_3\tau \int_{t-\tau}^{t} x^2(s)ds$$
$$+ \lambda_4\sigma^2x^2(t) - \lambda_4\sigma \int_{t-\sigma}^{t} x^2(s)ds + \lambda_5\delta^2x^2(t) - \lambda_5\delta \int_{t-\delta}^{t} x^2(s)ds$$
$$=(-2f_1(x)a(t) - 2\alpha - 2\beta + \lambda_1 + \lambda_2 + \lambda_3\tau^2 + \lambda_4\sigma^2 + \lambda_5\delta^2)x^2(t)$$
$$+ 2\alpha x(t)x(t-\tau) + 2\beta x(t)x(t-\sigma) - 2g_1(x(t-\sigma))b(t)x(t)x(t-\sigma)$$
$$- 2c(t)x(t) \int_{t-\delta}^{t} x(s)ds - 2(f_1(x)a(t) + \alpha + \beta)p(t)x(t)x(t-\tau)$$
$$+ 2\alpha p(t)x^2(t-\tau) + 2\beta p(t)x(t-\tau)x(t-\sigma)$$
$$- 2g_1(x(t-\sigma))b(t)p(t)x(t-\tau)x(t-\sigma) - 2p(t)c(t)x(t-\tau) \int_{t-\delta}^{t} x(s)ds$$
$$+ 2(f_1(x)a(t) + \alpha + \beta)\alpha x(t) \int_{t-\tau}^{t} x(s)ds - 2\alpha^2x(t-\tau) \int_{t-\tau}^{t} x(s)ds$$
$$- 2\alpha\beta x(t-\sigma) \int_{t-\tau}^{t} x(s)ds + 2\alpha g_1(x(t-\sigma))b(t)x(t-\sigma) \int_{t-\tau}^{t} x(s)ds$$
$$+ 2\alpha c(t) \int_{t-\tau}^{t} x(s)ds \int_{t-\delta}^{t} x(s)ds + 2(f_1(x)a(t) + \alpha + \beta)\beta x(t) \int_{t-\sigma}^{t} x(s)ds$$
$$- 2\alpha\beta x(t-\tau) \int_{t-\sigma}^{t} x(s)ds - 2\beta^2x(t-\sigma) \int_{t-\sigma}^{t} x(s)ds$$
$$+ 2\beta g_1(x(t-\sigma))b(t)x(t-\sigma) \int_{t-\sigma}^{t} x(s)ds + 2\beta c(t) \int\limits_{t-\sigma}^{t} x(s)ds \int_{t-\delta}^{t} x(s)ds$$

$$- \lambda_1 x^2(t-\tau) - \lambda_2 x^2(t-\sigma) - \lambda_3 \tau \int\limits_{t-\tau}^{t} x^2(s)ds - \lambda_4 \sigma \int\limits_{t-\sigma}^{t} x^2(s)ds$$

$$- \lambda_5 \delta \int_{t-\delta}^{t} x^2(s)ds.$$

By using hölder inequality we can easily see that

$$\begin{aligned} \tau \int_{t-\tau}^{t} x^2(s)ds &\geq \left(\int_{t-\tau}^{t} x(s)ds \right)^2, \\ \sigma \int_{t-\sigma}^{t} x^2(s)ds &\geq \left(\int_{t-\sigma}^{t} x(s)ds \right)^2, \\ \delta \int_{t-\delta}^{t} x^2(s)ds &\geq \left(\int_{t-\delta}^{t} x(s)ds \right)^2. \end{aligned}$$

Taking into account conditions 2.4 and 2.5, we have

$$\begin{aligned} \frac{dV}{dt} \leq & (-2m_1a_1 - 2\alpha - 2\beta + \lambda_1 + \lambda_2 + \lambda_3\tau^2 + \lambda_4\sigma^2 + \lambda_5\delta^2)x^2(t) \\ & + [2\alpha - 2(m_1a_1 + \alpha + \beta)p_1]x(t)x(t-\tau) + (2\beta - 2n_1b_1)x(t)x(t-\sigma) \\ & - 2c_1x(t) \int\limits_{t-\delta}^{t} x(s)ds + (2\alpha p_1 - \lambda_1)x^2(t-\tau) \\ & + (2\beta p_1 - 2n_1b_1p_1)x(t-\tau)x(t-\sigma) - 2p_1c_1x(t-\tau) \int_{t-\delta}^{t} x(s)ds \\ & + 2(m_2a_2\alpha + \alpha^2 + \alpha\beta)x(t) \int_{t-\tau}^{t} x(s)ds - 2\alpha^2 x(t-\tau) \int_{t-\tau}^{t} x(s)ds \\ & - (2\alpha\beta - 2\alpha n_2b_2)x(t-\sigma) \int_{t-\tau}^{t} x(s)ds + 2\alpha c_2 \int_{t-\tau}^{t} x(s)ds \int_{t-\delta}^{t} x(s)ds \\ & + 2\beta c_2 \int_{t-\sigma}^{t} x(s)ds \int_{t-\delta}^{t} x(s)ds + 2(m_2a_2\beta + \alpha\beta + \beta^2)x(t) \int_{t-\sigma}^{t} x(s)ds \\ & - 2\alpha\beta x(t-\tau) \int_{t-\sigma}^{t} x(s)ds - \lambda_2 x^2(t-\sigma) - (2\beta^2 - 2\beta n_2b_2)x(t-\sigma) \int_{t-\sigma}^{t} x(s)ds \\ & - \lambda_3 \left(\int_{t-\tau}^{t} x(s)ds \right)^2 - \lambda_4 \left(\int_{t-\sigma}^{t} x(s)ds \right)^2 - \lambda_5 \left(\int_{t-\delta}^{t} x(s)ds \right)^2. \end{aligned}$$

The last estimate implies that

$$\frac{dV}{dt} \leq \xi^T(t)\Pi\xi(t),$$

where $\xi^T(t) = \left[x(t) \quad x(t-\tau) \quad x(t-\sigma) \quad \int\limits_{t-\tau}^{t} x(s)ds \quad \int\limits_{t-\sigma}^{t} x(s)ds \quad \int\limits_{t-\delta}^{t} x(s)ds \right]$ and Π is defined in 2.7. Thus, 2.7 implied that there exists a positive constant $\mu > 0$ such that $\frac{dV}{dt} \leq -\mu \left\| D(x_t) \right\|$. Therefore, equation 2.6 is asymptotically stable according to [[8],Theorem 8.1, pp. 292–293]. This completes the proof.

Example 2.1. *Consider neutral differential equation 2.6 with*

$$a_1 = a_2 = 1, b_1 = b_2 = 0.5, c_1 = c_2 = 0, m_1 = m_2 = 2, n_1 = n_2 = 0.4, |p(t)| \leq p_1 = 0.25 < 1, \tag{2.8}$$

$$\tau = 0.2, \sigma = 0.4., \delta = 0.3, \alpha = 0.1, \beta = 0.3, \lambda_1 = 1.6, \lambda_2 = \lambda_3 = 1.2, , \lambda_4 = 0.8, \lambda_5 = 1.5. \tag{2.9}$$

Under the above assumptions, by solving matrix inequality 2.7 using Matlab, we found that the all eigenvalues of this matrix are -0.3125, -1.1539, -1.1931, -1.4085, -1.5000 and -2.3669. As a result, it is clear that all the conditions of Theorem 2.1 hold. This discussion implies that the zero solution of equation 2.6 is asymptotically stable.

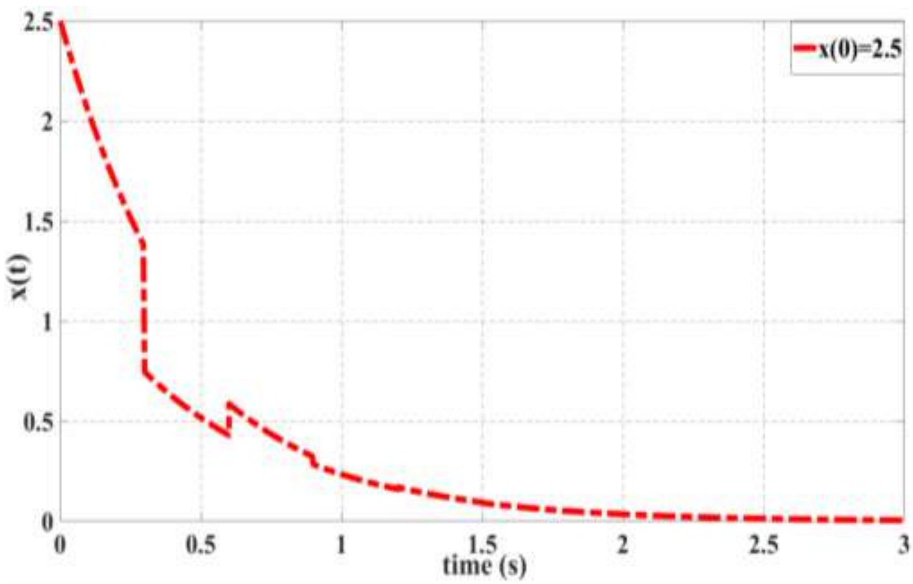

Figure 1. The simulation of the Example 2.1.

3. DTM and Numerical Experiment

The theory of DT can be found in [15, 16]. In this research paper, we will explain briefly. The DT of function $x(t)$ is defined as

$$X(k) = \frac{1}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \tag{3.10}$$

where $x(t)$is the original function and $X(k)$ is the transformed function.

Differential inverse transform of $X(k)$ is defined as

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}. \tag{3.11}$$

From 3.10 and 3.11, if the function $x(t)$ can be expressed in a finite series as follows

$$x(t) = \sum_{k=0}^{\infty} X(k)\, t^k = X(0) + X(1)\, t + X(2)\, t^2 + \ldots, \tag{3.12}$$

then it is called series solution of the DTM.

The following fundamental theorems can be easily deduced from equations 3.10 and 3.11 (also see,[17],[20]).

Theorem 3.1. *If* $x(t) = \frac{dx(t)}{dt}$*,then* $X(k) = \frac{(k+1)!}{k!} X(k+1) = (k+1)\, X(k+1)$.

Theorem 3.2. *If* $x(t) = \alpha x(t)$*, then* $X(k) = \alpha X(k)$*,where* α *is a constant.*

Theorem 3.3. *If* $x(t) = x(t-a)$*,* $a > 0$ *and reel constant, then*

$$X(k) = \sum_{i=k}^{N} (-1)^{i-k} \begin{pmatrix} i \\ k \end{pmatrix} a^{i-k} X(i).$$

Theorem 3.4. *If* $\frac{d}{dt} x(t-a)$*, then* $X(k) = (k+1) \sum_{i=k+1}^{N} (-1)^{i-k-1} \begin{pmatrix} i \\ k+1 \end{pmatrix} a^{i-k-1} X(i)$.

Theorem 3.5. *If* $x(t) = \int_{t_0}^{t} x(s) ds$*, then* $X(k) = \frac{X(k-1)}{k}$*,* $k \geq 1$*,* $X(0) = 0$.

Now, we demonstrate potentiality, advantages and effectiveness of our method on an example.

Example 3.1. *Under initial condition* $x(0) = 2.5$*, we consider the first order neutral differential equation 2.6 with 2.8 and 2.9. Taking into account Theorem 3.1 - 3.5, applying DTM on both sides of equation 3.10 and condition 3.11, we obtain the following recurrence relation*

$$\begin{aligned} X(0) =& 2.5, \\ (k+1)\, X(k+1) =& [-0.25\,(k+1) \sum_{i=k+1}^{N} (-1)^{i-k-1} \begin{pmatrix} i \\ k+1 \end{pmatrix} 0.2^{i-k-1} X(i) - 2X(k) \\ & - 0.2 \sum_{i=k}^{N} (-1)^{i-k} \begin{pmatrix} i \\ k \end{pmatrix} 0.4^{i-k} X(i)], \ k = 0, 1, ..., 6. \end{aligned}$$

Using this recurrence relation, the following series coefficients $X(k)$ *can be obtained. For* $N = 4$*,*

$X(1)$ =*-4.256423713,* $X(2)$=*4.173891756,* $X(3)$=*-3.190591724,* $X(4)$=*2.211301195,*

$X(5)$ =*-1.326780717,* $X(6)$=*0.4422602390,* $X(7)$=*-0.1263600683,* $k = 0, 1, ..., 6$.

For $N = 6$,

$X(1)$ =*-4.256931168,* $X(2)$=*4.169113047,* $X(3)$=*-3.134489650,* $X(4)$=*2.052537892,*

$X(5)$ =*-1.272263766,* $X(6)$=*0.7624052530,* $X(7)$=*-0.3703111229,* $k = 0, 1, ..., 6.$

For $N = 8$,

$X(1)$ =*-4.256957370,* $X(2)$=*4.169240023,* $X(3)$=*-3.133772360,* $X(4)$=*2.045844921,*

$X(5)$ =*-1.257197863,* $X(6)$=*0.7759998430,* $X(7)$=*-0.4899948359,* $k = 0, 1, ..., 6.$

Finally, using above mentioned relations, taking $N = 4, \quad 6, \quad 8$ *and using equation 3.12, we reach approximate solutions of equation 2.6 with 7 iterations as follows:*

$N = 4$,

$$x_{DTM}(t) = 2.5 - 4.256423713t + 4.173891756t^2 - 3.190591724t^3 + 2.211301195t^4$$
$$- 1.326780717t^5 + 4.422602390t^6 - 1.263600683t^7,$$

$N = 6$,

$$x_{DTM}(t) = 2.5 - 4.256931168t + 4.169113047t^2 - 3.134489650t^3 + 2.052537892t^4$$
$$- 1.272263766t^5 + 7.624052530t^6 - 3.703111229t^7,$$

$N = 8$,

$$x_{DTM}(t) = 2.5 - 4.256957370t + 4169240023t^2 - 3.133772360t^3 + 2.045844921t^4$$
$$- 1.257197863t^5 + 7.759998430t^6 - 4.899948359t^7.$$

As a result, it is seen that in the cases of $N = 4, \quad N = 6$ *and* $N = 8$, *our numerical results are almost the same.*

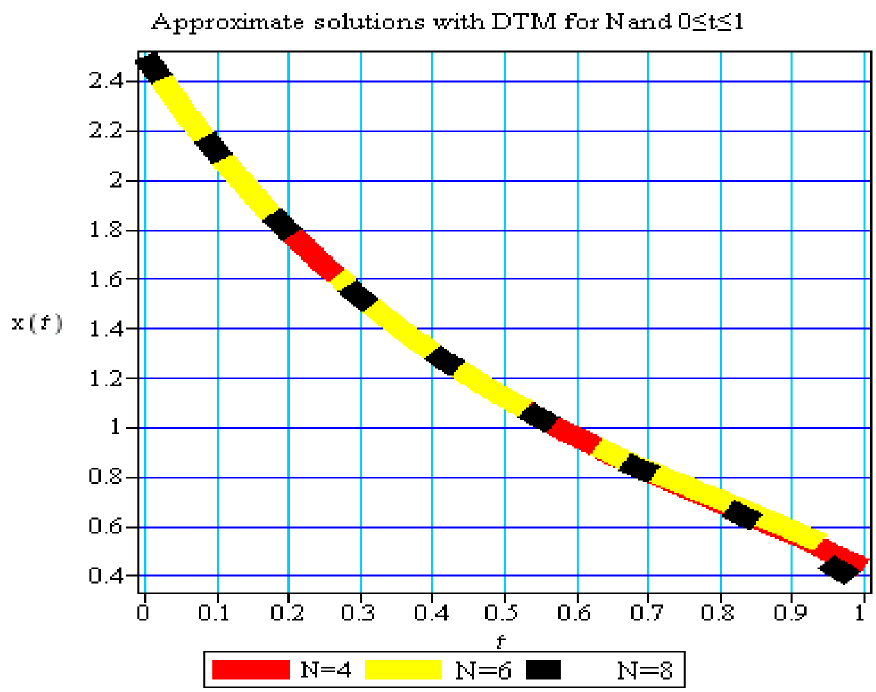

FIGURE 2. Comparison between approximate solutions using DTM.

TABLE 1. Comparison of numerical results obtained with DTM.

t	$N = 4$	$N = 6$	$N = 8$
0.0	2.5	2.5	2.5
0.1	2.113114246	2.113056779	2.113055630
0.2	1.793286393	1.793223362	1.793222389
0.3	1.527559403	1.527518365	1.527507432
0.4	1.305682872	1.305711365	1.305609636
0.5	1.119104674	1.119546373	1.118984564
0.6	0.960089977	0.961954349	0.959727281
0.7	0.820903961	0.826068104	0.819026077
0.8	0.692994495	0.703852936	0.684940092
0.9	0.566111176	0.584166389	0.539253944
1.0	0.427296967	0.450060485	0.353162358

4. CONCLUSIONS

In this study, we first derived some novel sufficient conditions to prove the asymptotic stability of solutions the first order neutral type differential equation. Subsequently, using

DTM, we obtained numerical approximations for different N ve t by an appropriate computer program. We constructed the Table 1 to make a comparison between the numerical results for $N = 4$, $N = 6$ and $N = 8$. By Matlab and an appropriate computer program, we provided two examples to show the effectiveness of proposed method. When the simulations of Example 2.1 and Example 3.1 are examined, the obtained results shows that the proposed methods are useful and applicable. As a suggestion, the techniques and methods presented for equation 1.1 can be improved with different situational or time-dependent delays.

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