



ON THE GEOMETRY OF LP-SASAKIAN MANIFOLDS ASSOCIATED WITH THE EXTENDED QUASI-CONFORMAL CURVATURE TENSOR

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Abstract. The geometry of LP-Sasakian manifolds and their interaction with various curvature tensors—particularly conformal, quasi-conformal, and concircular tensors—has been extensively studied in the literature. However, most existing works focus on classical curvature tensors or standard quasi-conformal curvature structures under general or well-known curvature restrictions. The present study introduces and investigates an extended quasi-conformal curvature tensor, which generalizes the traditional quasi-conformal curvature tensor by incorporating additional structural components adapted to LP-Sasakian manifolds.

In particular, we focus on LP-Sasakian manifolds with in the frame work of the extended quasi-conformal curvature tensor satisfying the vanishing conditions $K_e(U_1, Y_1)Z_1 = 0$ and $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$. Furthermore, the existence and properties of Ricci soliton structures are also examined.

Keywords: LP-Sasakian manifolds, extended quasi conformal curvature tensor, η -Einstein manifold, scalar curvature.

MSC (2020): 53C15, 53C25, 53C50.

1. INTRODUCTION

The extended quasi-conformal curvature tensor is a generalization of several well-known curvature tensors in differential geometry, designed to unify and extend notions like conformal, concircular, and quasi-conformal curvature. Its importance becomes clearer when you look at both its structural role and its applications in special manifolds, including LP-Sasakian manifolds. Its relevance becomes evident once we understand the limitations of the usual curvature measures.

In 1989, Matsumoto [8] first introduced the concept of LP-Sasakian manifolds. Subsequently, Mihai and Rosca [10] independently presented the same notion and established several significant results. Since then, LP-Sasakian manifolds have been extensively investigated by various

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researchers, including Bagewadi et al.,[3], De et al.,[4], Mihai et al.,[9], Shaikh et al.,[12, 14] and many others. Recently generalized Ricci-type solitons and almost Ricci-Bourguignon solitons on Lorentzian para-Kenmotsu geometry has been studied by [11, 15].

On the other hand, in [17] Yano and Sawaki introduced and investigated a tensor field K on an $(2n + 1)$ -dimensional Riemannian manifold that generalizes both the conformal curvature tensor and the concircular curvature tensor as particular instances. This tensor field K is called the quasi-conformal curvature tensor and is defined as follows:

$$\begin{aligned} K(U_1, Y_1)Z_1 &= aR(U_1, Y_1)Z_1 \\ &+ b[S(Y_1, Z_1)U_1 - S(U_1, Z_1)Y_1 + g(Y_1, Z_1)QU_1 - g(U_1, Z_1)QY_1] \\ &- \frac{r}{2n+1} \left[\frac{a}{2n} + 2b \right] \{g(Y_1, Z_1)U_1 - g(U_1, Z_1)Y_1\}, \end{aligned} \quad (1.1)$$

where each of U_1, Y_1, Z_1 belongs to the tangent space $T_p M$, while a and b are constants and r denotes the scalar curvature. In particular, when $a = 1$ and $b = -\frac{1}{n-2}$, then the quasi conformal curvature tensor coincides with the conformal curvature tensor.

Subsequently, De et al. [5] introduced and formulated an extended version of the quasi-conformal curvature tensor, referred to as the extended quasi-conformal curvature tensor, which is defined by:

$$\begin{aligned} K_e(U_1, Y_1)Z_1 &= K(U_1, Y_1)Z_1 - \eta(U_1)K(\xi, Y_1)Z_1 \\ &- \eta(Y_1)K(U_1, \xi)Z_1 - \eta(Z_1)K(U_1, Y_1)\xi. \end{aligned} \quad (1.2)$$

Recently, authors Al Hussein and Habeeb [1, 2] investigated and studied the geometric properties and applications of Quasi Sasakian manifolds endowed with generalized Sasakian space form structures in the framework of the pseudo-quasi conformal curvature tensor.

Motivated by the above, the present work investigates certain aspects of LP-Sasakian manifold admitting specific geometric properties associated with the extended quasi-conformal curvature tensor. The organization of the paper is as follows. After presenting the necessary preliminaries, Section 3 examines LP-Sasakian manifolds for which the extended quasi-conformal curvature tensor vanishes, that is, $K_e(U_1, Y_1)Z_1 = 0$. In Section 4, we analyze LP-Sasakian manifolds satisfying the condition $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$. The final section summarizes and concludes the findings established throughout the paper.

2. PRELIMINARIES

An $(2n + 1)$ -dimensional differentiable manifold M is said to be an LP-Sasakian manifold [8] if it possesses a $(1, 1)$ -tensor field ϕ , a unit time like vector field ξ , a 1-form η and a Lorentzian metric g that fulfill the following conditions:

$$\eta(\xi) = -1, \phi^2 U_1 = U_1 + \eta(U_1)\xi, \quad (2.3)$$

$$g(\phi U_1, \phi Y_1) = g(U_1, Y_1) + \eta(U_1)\eta(Y_1), \quad (2.4)$$

$$g(U_1, \xi) = \eta(U_1), \quad \nabla_{U_1} \xi = \phi U_1 \quad (2.5)$$

$$(\nabla_{U_1} \phi)(Y_1) = g(U_1, Y_1)\xi + \eta(Y_1)U_1 + 2\eta(U_1)\eta(Y_1)\xi, \quad (2.6)$$

for all vectors $U_1, Y_1 \in T_p M$, where ∇ represents the covariant derivative associated with the Lorentzian metric g .

It is readily observed that the following relations are satisfied in an LP-Sasakian manifold [8]:

$$\phi\xi = 0, \quad \eta(\phi U_1) = 0, \quad (2.7)$$

$$\text{rank}\phi = 2n. \quad (2.8)$$

As before, if we assume

$$\Omega(U_1, Y_1) = g(U_1, \phi Y_1) \text{ and } \nabla_{U_1}\xi = \phi U_1, \quad (2.9)$$

for arbitrary vector fields U_1, Y_1 , then the tensor $\Omega(U_1, Y_1)$ defines a symmetric $(0, 2)$ -type tensor field [8].

Moreover, since the vector field η is closed in an $(2n + 1)$ -dimensional LP-Sasakian manifold, it follows that [8, 10]:

$$(\nabla_{U_1}\eta)(Y_1) = \Omega(U_1, Y_1), \quad (2.10)$$

$$\Omega(U_1, \xi) = 0, \quad (2.11)$$

for any $U_1, Y_1 \in T_pM$.

Consider an $(2n + 1)$ -dimensional LP-Sasakian manifold M equipped with the structure (ϕ, ξ, η, g) . The following relations are satisfied [8]:

$$R(U_1, Y_1)Z_1 = g(Y_1, Z_1)U_1 - g(U_1, Z_1)Y_1, \quad (2.12)$$

$$R(\xi, Y_1)Z_1 = g(Y_1, Z_1)\xi - \eta(Z_1)Y_1, \quad (2.13)$$

$$R(U_1, Y_1)\xi = \eta(Y_1)U_1 - \eta(U_1)Y_1, \quad (2.14)$$

$$S(U_1, \xi) = 2n\eta(U_1), \quad (2.15)$$

$$S(\phi U_1, \phi Y_1) = S(U_1, Y_1) + 2n\eta(U_1)\eta(Y_1), \quad (2.16)$$

for any $U_1, Y_1, Z_1 \in T_pM$, where R is the Riemannian curvature tensor and S is the Ricci tensor of the manifold.

Additionally, for an $(2n + 1)$ -dimensional LP-Sasakian manifold with extended quasi-conformal curvature, the following holds:

$$\begin{aligned} K_e(U_1, Y_1)Z_1 = & \left[a - \frac{r}{2n+1} \left(\frac{a}{2n} + 2b \right) \right] \{ g(Y_1, Z_1)U_1 - g(U_1, Z_1)Y_1 \\ & + g(U_1, Z_1)\eta(Y_1)\xi - g(Y_1, Z_1)\eta(U_1)\xi + 2\eta(U_1)\eta(Z_1)Y_1 \\ & - 2\eta(Y_1)\eta(Z_1)U_1 \} + b \{ S(Y_1, Z_1)U_1 - S(U_1, Z_1)Y_1 \\ & + \eta(Y_1)S(U_1, Z_1)\xi - \eta(U_1)S(Y_1, Z_1)\xi + g(Y_1, Z_1)QU_1 \\ & - g(U_1, Z_1)QY_1 + 2\eta(U_1)\eta(Z_1)QY_1 - 2\eta(Y_1)\eta(Z_1)QU_1 \\ & + 2n\eta(Y_1)g(U_1, Z_1)\xi - 2n\eta(U_1)g(Y_1, Z_1)\xi \\ & + 4n\eta(U_1)\eta(Z_1)Y_1 - 4n\eta(Y_1)\eta(Z_1)U_1 \}, \end{aligned} \quad (2.17)$$

$$\begin{aligned}
K_e(\xi, Y_1)Z_1 &= \left[a - \frac{r}{2n+1} \left(\frac{a}{2n} + 2b \right) \right] \{ 2g(Y_1, Z_1)\xi - 3\eta(Z_1)Y_1 \\
&\quad - \eta(Y_1)\eta(Z_1)\xi \} + b[2S(Y_1, Z_1)\xi - 6n\eta(Z_1)Y_1 \\
&\quad + 4ng(Y_1, Z_1)\xi - 4n\eta(Y_1)\eta(Z_1)\xi - 3\eta(Z_1)QY_1] \\
&= -K_e(Y_1, \xi)Z_1,
\end{aligned} \tag{2.18}$$

$$\begin{aligned}
K_e(U_1, Y_1)\xi &= \left[a - \frac{r}{2n+1} \left(\frac{a}{2n} + 2b \right) \right] [3\eta(Y_1)U_1 - 3\eta(U_1)Y_1] \\
&\quad + b[6n\eta(Y_1)U_1 - 6n\eta(U_1)Y_1 + 3\eta(Y_1)QU_1 - 3\eta(U_1)QY_1],
\end{aligned} \tag{2.19}$$

$$K_e(\xi, \xi)Z_1 = 0. \tag{2.20}$$

Definition 2.1. An $(2n+1)$ -dimensional LP-Sasakian manifold is called η -Einstein, if its Ricci tensor S of type $(0, 2)$ satisfies

$$S(U_1, Y_1) = \alpha g(U_1, Y_1) + \beta \eta(U_1)\eta(Y_1), \tag{2.21}$$

for all $U_1, Y_1 \in T_p M$, where α and β are smooth functions. Moreover, if $\beta = 0$, the manifold is referred to as an Einstein manifold.

Definition 2.2. In an $(2n+1)$ -dimensional LP-Sasakian manifold, if the Ricci tensor S satisfies

$$(\nabla_{W_1} S)(\phi U_1, \phi Y_1) = 0, \tag{2.22}$$

for all $U_1, Y_1, W_1 \in T_p M$, then S is called η -parallel.

A Ricci soliton extends the concept of an Einstein metric and is formulated on a Riemannian manifold (M, g) [6]. Specifically, a Ricci soliton is represented by a triple (g, ξ, λ) consisting of a Riemannian metric g , a unit timelike vector field ξ , and a real scalar λ , satisfying

$$L_V g + 2S + 2\lambda g = 0. \tag{2.23}$$

Here S denotes the Ricci tensor and L_V represents the Lie derivative taken along the solenoidal vector field V .

A Ricci soliton on a Riemannian manifold is classified as shrinking, steady, or expanding depending on whether the scalar λ is negative, zero, or positive, respectively.

3. LP-SASAKIAN MANIFOLDS ADMITTING VANISHING EXTENDED QUASI-CONFORMAL CURVATURE TENSOR

Consider an $(2n+1)$ -dimensional LP-Sasakian manifold in which the extended quasi-conformal curvature tensor is identically zero i.e.,

$$K_e(U_1, Y_1)Z_1 = 0, \tag{3.24}$$

for any choice of U_1, Y_1, Z_1 in the tangent space $T_p M$.

It follows directly from (3.24) that

$$K(U_1, Y_1)Z_1 - \eta(U_1)K(\xi, Y_1)Z_1 - \eta(Y_1)K(U_1, \xi)Z_1 - \eta(Z_1)K(U_1, Y_1)\xi = 0. \tag{3.25}$$

Using (3.25) and then taking the inner product with W_1 , we obtain

$$\begin{aligned} & \left[a - \frac{r}{2n+1} \left(\frac{a}{2n} + 2b \right) \right] [g(Y_1, Z_1)g(U_1, W_1) - g(U_1, Z_1)g(Y_1, W_1) \\ & - \eta(U_1)\eta(W_1)g(Y_1, Z_1) + \eta(Y_1)\eta(W_1)g(U_1, Z_1) + 2\eta(U_1)\eta(Z_1)g(Y_1, W_1) \\ & - 2\eta(Y_1)\eta(Z_1)g(U_1, W_1)] + b[S(Y_1, Z_1)g(U_1, W_1) - S(U_1, Z_1)g(Y_1, W_1) \\ & + g(Y_1, Z_1)S(U_1, W_1) - g(U_1, Z_1)S(Y_1, W_1) - \eta(U_1)\eta(W_1)S(Y_1, Z_1) \\ & + \eta(Y_1)\eta(W_1)S(U_1, Z_1) + 4n\eta(U_1)\eta(Z_1)g(Y_1, W_1) - 4n\eta(Y_1)\eta(Z_1)g(U_1, W_1) \\ & - 2n\eta(U_1)\eta(W_1)g(Y_1, Z_1) + 2n\eta(Y_1)\eta(W_1)g(U_1, Z_1) \\ & + 2\eta(U_1)\eta(Z_1)S(Y_1, W_1) - 2\eta(Y_1)\eta(Z_1)S(U_1, W_1)] = 0. \end{aligned} \quad (3.26)$$

By substituting $U_1 = W_1 = e_i$, where e_i forms an orthonormal basis for the tangent space at each point on the manifold, and summing over i , $i = 1, 2, \dots, 2n+1$, we get

$$\begin{aligned} & \left[a - \frac{r}{2n+1} \left(\frac{a}{2n} + 2b \right) \right] [2ng(Y_1, Z_1) - g(Y_1, Z_1) + g(Y_1, Z_1) \\ & + \eta(Y_1)\eta(Z_1) + 2\eta(Y_1)\eta(Z_1) - 4n\eta(Y_1)\eta(Z_1)] + b[2nS(Y_1, Z_1) \\ & - S(Y_1, Z_1) + rg(Y_1, Z_1) - S(Y_1, Z_1) + S(Y_1, Z_1) + 2n\eta(Y_1)\eta(Z_1) \\ & + 4n\eta(Y_1)\eta(Z_1) - 8n^2\eta(Y_1)\eta(Z_1) + 2ng(Y_1, Z_1) + 2n\eta(Y_1)\eta(Z_1) \\ & + 4n\eta(Y_1)\eta(Z_1) - 2r\eta(Y_1)\eta(Z_1)] = 0. \end{aligned} \quad (3.27)$$

The equation above can be transformed into

$$S(Y_1, Z_1) = l_1g(Y_1, Z_1) + m_1\eta(Y_1)\eta(Z_1), \quad (3.28)$$

where

$$\begin{aligned} l_1 &= \frac{a2n(r - 2n - 1) + b2n[r(2n - 1) - 2n(2n + 1)]}{b2n(2n + 1)(2n - 1)}, \\ m_1 &= \frac{a(4n - 3)[2n(2n + 1) - r] + b4n[r(-2n + 4) - (2n + 1)(4n^2 - 6n)]}{b2n(2n + 1)(2n - 1)}. \end{aligned}$$

Thus we can state the following:

Theorem 3.1. *If an LP-Sasakian manifold has a vanishing extended quasi-conformal curvature tensor, then it is an η -Einstein manifold.*

Applying $Y_1 = Z_1 = e_i$ once again in (3.28) results in

$$r = \frac{a(2n - 1) - b(2n - 3)}{a(2n + 1)(2n - 1) + b2n(2n + 1)(2n - 2)}. \quad (3.29)$$

This leads to the following:

Corollary 3.1. *In an LP-Sasakian manifold where the extended quasi-conformal curvature tensor vanishes, the scalar curvature remains constant.*

Similarly, by replacing Y_1 by ϕY_1 and Z_1 by ϕZ_1 in (3.28) and then using (2.5), we obtain

$$S(\phi Y_1, \phi Z_1) = \frac{a(r - 2n - 1) + b[r(2n - 1) - 2n(2n + 1)]}{b(2n + 1)(2n - 1)}g(\phi Y_1, \phi Z_1). \quad (3.30)$$

Applying covariant differentiation to (3.30) with respect to X_1 , we arrive at

$$(\nabla_{X_1} S)(\phi Y_1, \phi Z_1) = dr(X_1) \frac{a + b(2n - 1)}{b(2n + 1)(2n - 1)} g(\phi Y_1, \phi Z_1). \quad (3.31)$$

Given that an LP-Sasakian manifold with a vanishing extended quasi-conformal curvature tensor has constant scalar curvature, it follows from the above equation that

$$(\nabla_{X_1} S)(\phi Y_1, \phi Z_1) = 0. \quad (3.32)$$

This results in:

Corollary 3.2. *An LP-Sasakian manifold with vanishing extended quasi-conformal curvature tensor and constant scalar curvature possesses an η -parallel Ricci tensor.*

Taking an account of (3.28) in (2.23), we get

$$\begin{aligned} L_V g(Y_1, Z_1) + \frac{a2n(r - 2n - 1) + b2n[r(2n - 1) - 2n(2n + 1)]}{b2n(2n + 1)(2n - 1)} 2g(Y_1, Z_1) \\ + \frac{a(4n - 3)[2n(2n + 1) - r] + b4n[r(-2n + 4) - n(8n^2 - 8n - 6)]}{b2n(2n + 1)(2n - 1)} 2\eta(Y_1)\eta(Z_1) \\ + 2\lambda g(Y_1, Z_1) = 0. \end{aligned} \quad (3.33)$$

By setting $Y_1 = Z_1 = e_i$, the above equation becomes

$$\begin{aligned} \operatorname{div}(V) + \lambda b8n^2(2n + 1)(2n - 1) \\ + 2a(2n - 1)[r(2n + 3) - 6n(2n + 1)] \\ + 4bn[r(4n^2 + 2n - 8) - 3(2n + 1)(4n^2 - 4n)] = 0. \end{aligned} \quad (3.34)$$

Because V is a solenoidal vector field, it follows that

$$\lambda = - \left\{ \frac{a(2n - 1)[r(2n + 3) - 6n(2n + 1)] + b2n[r(4n^2 + 2n - 8) - 3(2n + 1)(4n^2 - 4n)]}{b4n^2(2n + 1)(2n - 1)} \right\} \quad (3.35)$$

Consequently, the following occurs:

Remark 3.1. *In an LP-Sasakian manifold with a vanishing extended quasi-conformal curvature tensor, the Ricci soliton is of shrinking type.*

4. LP-SASAKIAN MANIFOLDS ADMITTING EXTENDED QUASI-CONFORMAL CURVATURE TENSOR SATISFYING $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$

Let M be an $(2n + 1)$ -dimensional LP-Sasakian manifold for which the curvature condition $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$ holds. The preceding condition implies that

$$\begin{aligned} g(Y_1, K_e(U_1, V_1)W_1)\xi - \eta(K_e(U_1, V_1)W_1)Y_1 - g(Y_1, U_1)K_e(\xi, V_1)W_1 \\ + \eta(U_1)K_e(Y_1, V_1)W_1 - g(Y_1, V_1)K_e(U_1, \xi)W_1 + \eta(V_1)K_e(U_1, Y_1)W_1 \\ - g(Y_1, W_1)K_e(U_1, V_1)\xi + \eta(W_1)K_e(U_1, V_1)Y_1 = 0. \end{aligned} \quad (4.36)$$

In view of (4.36), we have

$$\begin{aligned} & -g(Y_1, K_e(U_1, V_1)W_1) - \eta(K_e(U_1, V_1)W_1)\eta(Y_1) - g(Y_1, U_1)\eta(K_e(\xi, V_1)W_1) \quad (4.37) \\ & + \eta(U_1)\eta(K_e(Y_1, V_1)W_1) + \eta(W_1)\eta(K_e(U_1, V_1)Y_1) + \eta(V_1)\eta(K_e(U_1, Y_1)W_1) \\ & - g(Y_1, W_1)\eta(K_e(U_1, V_1)\xi) - g(Y_1, V_1)\eta(K_e(U_1, \xi)W_1) = 0. \end{aligned}$$

Making use of (2.17)-(2.20) in (4.37), we derive

$$\begin{aligned} & \left[a - \frac{r}{2n+1} \left(\frac{a}{2n} + 2b \right) \right] [g(V_1, W_1)g(Y_1, U_1) - g(U_1, W_1)g(Y_1, V_1) \quad (4.38) \\ & + g(V_1, W_1)\eta(U_1)\eta(Y_1) - g(U_1, W_1)\eta(V_1)\eta(Y_1) + 2g(Y_1, U_1)\eta(V_1)\eta(W_1) \\ & - 2g(V_1, Y_1)\eta(U_1)\eta(W_1)] + b[S(V_1, W_1)g(Y_1, U_1) - S(U_1, W_1)g(V_1, Y_1) \\ & + S(V_1, Y_1)g(U_1, W_1) - S(Y_1, U_1)g(V_1, W_1) + S(V_1, W_1)\eta(U_1)\eta(Y_1) \\ & - S(U_1, W_1)\eta(V_1)\eta(Y_1) + 2ng(V_1, W_1)\eta(U_1)\eta(Y_1) - 2ng(U_1, W_1)\eta(V_1)\eta(Y_1) \\ & + 8ng(Y_1, U_1)\eta(V_1)\eta(W_1) - 8ng(Y_1, V_1)\eta(U_1)\eta(W_1) + 4ng(V_1, W_1)g(Y_1, U_1) \\ & - 4ng(U_1, W_1)g(V_1, Y_1)] = 0. \end{aligned}$$

Putting $U_1 = W_1 = e_i$, where e_i is an orthonormal basis of the tangent space at each point of the manifold, and taking summation over i , $i = 1, 2, \dots, 2n + 1$, we obtain

$$\begin{aligned} & \left[a - \frac{r}{2n+1} \left(\frac{a}{2n} + 2b \right) \right] [g(Y_1, V_1) - 2ng(Y_1, V_1) + \eta(Y_1)\eta(V_1) \quad (4.39) \\ & - 2n\eta(Y_1)\eta(V_1) + 2\eta(Y_1)\eta(V_1) + 2g(Y_1, V_1)] + b[S(Y_1, V_1) \\ & - rg(Y_1, V_1) + 2nS(V_1, Y_1) - S(Y_1, V_1) + 2n\eta(Y_1)\eta(V_1) \\ & - r\eta(Y_1)\eta(V_1) + 2n\eta(Y_1)\eta(V_1) - 4n^2\eta(Y_1)\eta(V_1) + 8ng(Y_1, V_1) \\ & + 8n\eta(Y_1)\eta(V_1) + 4ng(Y_1, V_1) - 8n^2g(Y_1, V_1)] = 0. \end{aligned}$$

The preceding equation can be simplified as

$$S(Y_1, V_1) = l_1g(Y_1, V_1) + m_1\eta(Y_1)\eta(V_1), \quad (4.40)$$

where

$$\begin{aligned} l_2 &= \frac{a(2n-3)[2n(2n+1)-r] + b2n[4n(2n+1)(2n-3) - r(2n-7)]}{b4n^2(2n+1)}, \\ m_2 &= \frac{a(2n-3)[2n(2n+1)-r] + b2n[2n(2n-6) - r(2n-7)]}{b4n^2(2n+1)}. \end{aligned}$$

Therefore, we can conclude that:

Theorem 4.1. *An $(2n + 1)$ -dimensional LP-Sasakian manifold that admits an extended quasi-conformal curvature tensor and satisfies $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$ is an η -Einstein manifold.*

Plugging $Y_1 = V_1 = e_i$ again into (4.40) gives

$$r = \frac{a2n(2n+1)(2n-1) + b8n^2[2n(2n+1)(2n-3) - (2n-)]}{a(2n-1)(2n-3) + b2n[2n(2n+1) + (2n-1)(2n-7)]}. \quad (4.41)$$

As a result, we can assert that:

Corollary 4.1. *If an LP-Sasakian manifold has an extended quasi-conformal curvature tensor satisfies $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$, then its scalar curvature is constant.*

Now replacing Y_1 by ϕY_1 and V_1 by ϕV_1 in (4.40), gives

$$S(\phi Y_1, \phi V_1) = Ag(\phi Y_1, \phi V_1). \quad (4.42)$$

where,

$$A = \frac{a(2n-3)[2n(2n+1)-r] + b2n[4n^2(2n-5) - r(2n-7)]}{b4n^2(2n+1)}.$$

Computing the covariant derivative of (4.42) along X_1 , we find

$$(\nabla_{X_1} S)(\phi Y_1, \phi V_1) = \frac{[-a(2n-3) - b2n(2n-7)]dr(X_1)}{b4n^2(2n+1)}g(\phi Y_1, \phi V_1). \quad (4.43)$$

Since an LP-Sasakian manifold endowed with an extended quasi-conformal curvature tensor and satisfying $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$ has constant scalar curvature, it follows that

$$(\nabla_{X_1} S)(\phi Y_1, \phi V_1) = 0. \quad (4.44)$$

From this, it follows that:

Corollary 4.2. *An LP-Sasakian manifold whose extended quasi-conformal curvature tensor satisfies $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$ necessarily has a Ricci tensor that is η -parallel.*

Upon substituting (4.40) into (2.23), we arrive at

$$\begin{aligned} & L_V g(V_1, Y_1) \\ & + \frac{a(2n-3)[2n(2n+1)-r] + b2n[4n(2n+1)(2n-3) - r(2n-7)]}{b4n^2(2n+1)} 2g(V_1, Y_1) \\ & + \frac{a(2n-3)[2n(2n+1)-r] + b2n[2n(2n-6) - r(2n-7)]}{b4n^2(2n+1)} 2\eta(V_1)\eta(Y_1) \\ & + 2\lambda g(V_1, Y_1) = 0. \end{aligned} \quad (4.45)$$

If we substitute $V_1 = Y_1 = e_i$ into (4.45), we obtain

$$\begin{aligned} & \text{div}(V) + \lambda b16n^3(2n+1) + a2(2n+1)(2n-3)[2n(2n+1)-r] \\ & + b4n[2n(8n^3 - 16n^2 - 5n - 3) - r(2n+1)(2n-7)] = 0. \end{aligned} \quad (4.46)$$

Owing to the fact that V is solenoidal, we get

$$\lambda = - \left\{ \frac{a(2n+1)(2n-3)[2n(2n+1)-r] + b2n[2n(8n^3 - 16n^2 - 5n - 3) - r(2n+1)(2n-7)]}{b8n^3(2n+1)} \right\} \quad (4.47)$$

This leads us to conclude that:

Remark 4.1. *On an LP-Sasakian manifold whose extended quasi-conformal curvature tensor satisfies $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$, the Ricci soliton is shrinking.*

5. Conclusion:

In general, the extended form of the quasi-conformal curvature tensor aligns with the extended quasi-conformal curvature tensor defined by De and Sarkar [5]. In this work, we investigate an n -dimensional LP-Sasakian manifold equipped with the extended quasi-conformal curvature tensor. Imposing the curvature conditions $K_e(U_1, Y_1)Z_1 = 0$ and $(R(\xi, Y_1) \cdot K_e)(U_1, V_1)W_1 = 0$, we derive the following geometric results:

- The LP-Sasakian manifold to take on a highly ordered, Einstein-type configuration.
- The manifold has uniform curvature distribution at every point.
- The Ricci tensor is stable along the contact distribution.
- The geometry of a manifold does not remain fixed or increase—it diminishes over time under Ricci flow.

Future research directions:

- Consider non-vanishing or bounded cases, this would help determine whether the Einstein-type configuration obtained is stable or if small perturbations lead to entirely different geometric behavior.
- Examining analogous results in Sasakian, Kenmotsu, or trans-Sasakian manifolds would show whether the conclusions depend heavily on the LP-structure or reflect a broader phenomenon in contact geometry.

REFERENCES

- [1] AlHussein, F.H., Habeeb, M.A. (2025). Quasi Sasakian manifold endowed with vanishing pseudo quasi conformal curvature tensor. *MethodsX*, 15, 1-11.
- [2] AlHussein, F.H., Habeeb, M.A. (2025). Pseudo-Quasi Conformal Curvature Tensor of Quasi Sasakian Manifold with Generalized Sasakian Space Forms. *Malaysian Journal of Mathematical Sciences*, 19(2), 383-398.
- [3] Bagewadi, C.S., Venkatesha, & Basavarajappa, N.S. (2008). On LP-Sasakian manifolds. *Sci. Scr. A: Math. Sci.*, 16, 1–8.
- [4] De, U.C., Matsumoto, K., & Shaikh, A.A. (1999). On Lorentzian Para-Sasakian manifolds. *Rendicontidel Seminario Matemetico di Messina, Serien II, Supplemto al.*, 3, 149–156.
- [5] De, U.C., & Sarkar, A. (2012). On the quasi conformal curvature tensor of a (K, μ) contact metric manifold. *Math. Reports*, 14(02), 115–129.
- [6] Hamilton, R.S. (1988). The Ricci flow on surfaces. In Mathematics and general relativity. *Amer. Math. Soc., Providence*, 71, 237–262.
- [7] Kobayashi, S., & Nomizu, K. (1963). Foundations of differential geometry. *Interscience Publishers*, 01, 293–301.
- [8] Matsumoto, K. (1989). On Lorentzian almost paracontact manifolds. *Bull. Yamagata Univ. Natur. Sci.*, 12(02), 151–156.
- [9] Mihai, I., & De, U.C. (1999). On Lorentzian Para-Sasakian manifolds. *Korean J. Math.*, 06, 1–13.
- [10] Mihai, I., & Rosca, R. (1992). On Lorentzian Para-Sasakian manifolds. *Classical Analysis, World Scientific Publ., Singapore*, 155–169.
- [11] Naveen Kumar, R. T., Siddesha, M. S., Swapna Sangeetha, P., Sowmyashree, B. M., & Siva Kota Reddy, P. (2025). Characterization of generalized Ricci-type solitons on Lorentzian para-Kenmotsu manifolds. *Dynamics of Continuous, Discrete and Impulsive Systems Series A: Mathematical Analysis*, 32, 271–281.
- [12] Shaikh, A.A., & Baishya, K.K. (2006). Some results on LP-Sasakian manifolds. *Bull. Math. Soc. Sci. Math., Roumanic Tome*, 49(97), 193–205.
- [13] Shaikh, A.A., & Biswas, S. (2004). On LP-Sasakian manifolds. *Bull. Malaysian Math. Sci. Soc.*, 2, 17–26.

- [14] Shaikh, A.A., Baishya, K. K., & Eyasmin, S. (2008). On the existence of some types of LP -Sasakian manifolds. *Commun. Korean Math. soc.*, 23(01), 1–16.
- [15] Swapna Sangeetha P, & Siddesha, M. S. (2026). On the structure of almost Ricci-Bourguignon solitons in Lorentzian para-Kenmotsu geometry. *Calculation*, 2(01), 2–11.
- [16] Yano, K., & Kon, M. (1984). Structures on manifolds. *Series in pure Math, Vol. 3, World scientific publ. co., Singapore*.
- [17] Yano, K., & Sawaki, S. (1968). Riemannian manifolds admitting a conformal transformation group. *J. Diff. Geom.*, 02, 161-184.

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