



NONEXISTENCE OF GLOBAL SOLUTIONS TO SEMI-LINEAR FRACTIONAL EVOLUTION EQUATION

MEDJAHED DJILALI* AND ALI HAKEM

ABSTRACT. In this paper, we consider the following semi-linear fractional evolution equation

$$u_{tt} + (-\Delta)^{\frac{\beta}{2}} u + D_{0|t}^{\alpha} u = h(t, x) |u|^p,$$

posed in $(0, T) \times \mathbb{R}^N$, where $(-\Delta)^{\frac{\beta}{2}}$, $0 < \beta \leq 2$ is $\frac{\beta}{2}$ - fractional power of $-\Delta$, and $D_{0|t}^{\alpha}$ denotes the derivatives of order α in the sense of Caputo. The nonexistence of global solutions theorem is established. Our method of proof is based on suitable choices of the test functions in the weak formulation of the sought solutions.

1. INTRODUCTION

Our main interest lies in the following problem:

$$\left\{ \begin{array}{l} u_{tt} + (-\Delta)^{\frac{\beta}{2}} u + D_{0|t}^{\alpha} u = h(t, x) |u|^p, \quad (t, x) \in (0, +\infty) \times \mathbb{R}^N \\ u(0, x) = u_0(x) \geq 0, \quad u_t(0, x) = u_1(x) \geq 0, \quad x \in \mathbb{R}^N, \end{array} \right. \quad (1.1)$$

where $p > 1, 0 < \alpha < 1, 0 < \beta \leq 2$ are constants. The function h is a non-negative and assumed to satisfy the condition

Received:2018-10-17

Accepted:2018-12-01

2010 Mathematics Subject Classification: 47J35, 35A01, 35R11, 35D30, 26A33.

Key words: Fractional Laplacian, fractional derivative, test function.

* Corresponding author

$$h(t, x) \geq Ct^\nu |x|^\mu, \text{ where } C > 0, \nu \geq 0, \mu \geq 0. \quad (1.2)$$

$D_{0/t}^\alpha$ denotes the derivatives of order α in the sense of Caputo and $(-\Delta)^{\frac{\beta}{2}}$ is the fractional power of $(-\Delta)$. The integral representation of the fractional Laplacian in the N -dimensional space [17] is

$$(-\Delta)^{\beta/2}\psi(x) = -c_N(\beta) \int_{\mathbb{R}^N} \frac{\psi(x+z) - \psi(x)}{|z|^{N+\beta}} dz, \quad \forall x \in \mathbb{R}^N, \quad (1.3)$$

where $c_N(\beta) = \Gamma((N+\beta)/2)/(2\pi^{N/2+\beta}\Gamma(1-\beta/2))$ and Γ denotes the gamma function. Note that the fractional Laplacian $(-\Delta)^{\beta/2}$ with $\beta \in (0, 2]$ is a pseudo-differential operator defined by:

$$(-\Delta)^{\beta/2}u(x) = \mathcal{F}^{-1} [|\zeta|^\beta \mathcal{F}(u)(\zeta)](x) \text{ for all } x \in \mathbb{R}^N,$$

where \mathcal{F} and \mathcal{F}^{-1} are Fourier transform and its inverse, respectively. Let us point out that many authors investigated the cases where $\alpha = 1, \beta = 2$ in several contexts. For example, the following Cauchy problem:

$$\begin{cases} u_{tt} + u_t - \Delta u = |u|^p, & (t, x) \in (0, \infty) \times (\mathbb{R}^N) \\ u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), & x \in \mathbb{R}^N, \end{cases} \quad (1.4)$$

has been investigated by Qi. Zhang [20] in the case $1 < p < 1 + \frac{2}{N}$, when $u_i, i = 0, 1$ is compactly supported and $\int u_i(x)dx > 0$. He proved that the solution of (1.4) does not exist globally. Therefore, he showed that $p = 1 + \frac{2}{N}$ belongs to the blow-up case.

Ogawa and H. Takeda [13] studied (1.4) as a initial boundary value problem in an exterior domain Ω . They established the non-existence of non-negative global solutions of the above problem when $1 < p < 1 + \frac{2}{N}$ and the initial data $(u_0, u_1) \in H_0^1(\Omega) \times L^2(\Omega)$ having a compact support. After that Fino and Wehbe [3] generalized the results of Ogawa-Takeda [13] by proving the blow-up of solutions of (1.4) under weaker assumptions on the initial data and they extended this results to the critical case $p = 1 + \frac{2}{N}$.

Todorova-Yordanov [19] showed that, if $p_c < p \leq \frac{N}{N-2}$, for $n \geq 3$ and $p_c < p < +\infty$, for $N = 1, 2$, where $p_c = 1 + \frac{2}{N}$, then (1.4) subjected to initial data $u(0, x) = \epsilon u_0(x)$, $u_t(0, x) = \epsilon u_1(x)$, $\epsilon > 0, x \in \mathbb{R}^N$, admits a unique global solution, and they proved that if $1 < p < 1 + \frac{2}{N}$, then the solution u blows up in a finite time. R. Ikehata [10], subjected the problem (1.4) with initial-boundary values, he derived certain decay estimates for the total energy of the

solution to the problem (1.4), when $1 + \frac{4}{N+2} < p \leq \frac{N}{N-1}$. In particular, A. Hakem [8] treated the following problem:

$$\begin{cases} u_{tt} + g(t)u_t - \Delta u = |u|^p, & (t, x) \in (0, \infty) \times (\mathbb{R}^N), \\ u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), & x \in \mathbb{R}^N, \end{cases} \quad (1.5)$$

where $g(t)$ is a function behaving like t^β , $0 \leq \beta < 1$. He obtained the non-existence of weak solution for the problem (1.5), when $1 < p \leq \frac{N+2}{N+2\beta}$.

Our purpose of this work is to generalize some of the above results, so with the suitable choice of the test function, we prove the non-existence of nontrivial global weak solution of (1.1).

2. PRELIMINARIES

Set $\Sigma_T = (0, T) \times (\mathbb{R}^N)$. The results of our research are based on the following definitions:

Definition 2.1. Let $0 < \alpha < 1$ and $\zeta' \in L^1(0, T)$. The left-sided and respectively right-sided Caputo derivatives of order α for ζ are defined as:

$$D_{0|t}^\alpha \zeta(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\zeta'(s)}{(t-s)^\alpha} ds \quad \text{and} \quad D_{t|T}^\alpha \zeta(t) = -\frac{1}{\Gamma(1-\alpha)} \int_t^T \frac{\zeta'(s)}{(s-t)^\alpha} ds,$$

where Γ denotes the gamma function (see [14] p 79).

Definition 2.2. We say that $u \geq 0$ is a local weak solution to (1.1), defined in Σ_T , $0 < T < +\infty$, if u is a locally integrable function such that $u^p h \in L_{loc}^1(\Sigma_T)$ and

$$\begin{aligned} & \int_{\Sigma_T} h |u|^p \Psi \, dx \, dt + \int_{\mathbb{R}^N} u_0(x) \Psi(0, x) \, dx + \int_{\mathbb{R}^N} u_1(x) \Psi(0, x) \, dx - \int_{\mathbb{R}^N} u_0(x) \Psi_t(0, x) \, dx \\ &= \int_{\Sigma_T} u \Psi_{tt} \, dx \, dt + \int_{\Sigma_T} u D_{t|T}^\alpha \Psi \, dx \, dt + \int_{\Sigma_T} u (-\Delta)^{\frac{\beta}{2}} \Psi \, dx \, dt, \end{aligned}$$

is satisfied for any $\Psi \in C_{t,x}^{2,2}(\Sigma_T)$ such that $\Psi(T, \cdot) = \Psi_t(T, \cdot) = 0$.

Definition 2.3. We say that $u \geq 0$ is global weak solution to (1.1) if it is a local solution to (1.1) defined in Σ_T for any $T > 0$.

Now, we recall the following integration by parts formula (see [18] p 46):

$$\int_0^T \phi(t) (D_{0|t}^\alpha \psi)(t) dt = \int_0^T (D_{t|T}^\alpha \phi)(t) \psi(t) dt. \quad (2.6)$$

We notice that, in all steps of proof, $C > 0$ is a real positive number which may change from line to line.

3. MAIN RESULTS

Our main result reads as follows:

Theorem 3.1. *Assume that $p > 1, 0 < \alpha < 1, 0 < \beta \leq 2$ and the conditions (1.2) are satisfied, if*

$$p \leq \frac{\alpha(N + \mu) + \beta(1 + \nu)}{\alpha N + \beta(1 - \alpha)}, \quad (3.7)$$

then the problem (1.1) has no nontrivial global weak solutions.

Proof. Since the principle of the method is the right choice of the test function, we choose it as follows:

$$\Psi(t, x) = \Phi\left(\frac{t^2 + |x|^{\frac{2\beta}{\alpha}}}{R^2}\right), \quad R > 0,$$

where Φ is a cut-off no increasing function satisfying

$$\Phi(r) = \begin{cases} 0, & \text{if } r \geq 2, \\ 1, & \text{if } r \leq 1, \end{cases}$$

and

$$0 \leq \Phi \leq 1, \quad \text{for all } r > 0.$$

Now multiplying the equation (1.1) by Ψ and integrating by parts on $\Sigma_T = (0, T) \times (\mathbb{R}^N)$, we get

$$\begin{aligned} & \int_{\Sigma_T} h |u|^p \Psi \, dx \, dt + \int_{\mathbb{R}^N} u_0(x) \Psi(0, x) \, dx + \int_{\mathbb{R}^N} u_1(x) \Psi(0, x) \, dx \\ & - \int_{\mathbb{R}^N} u_0(x) \Psi_t(0, x) \, dx = \int_{\Sigma_T} u \Psi_{tt} \, dx \, dt - \int_{\Sigma_T} u D_{0|t}^\alpha \Psi \, dx \, dt + \int_{\Sigma_T} u (-\Delta)^{\frac{\beta}{2}} \Psi \, dx \, dt. \end{aligned} \quad (3.8)$$

Invoking the fact that

$$\Psi_t(t, x) = 2tR^{-2} \Phi'\left(\frac{t^2 + |x|^{\frac{2\beta}{\alpha}}}{R^2}\right),$$

we easily deduce that $\Psi_t(0, x) = 0$. By using (2.6), the formula (3.8) will be on the shape

$$\begin{aligned} & \int_{\Sigma_T} h |u|^p \Psi \, dx \, dt + \int_{\mathbb{R}^N} u_0(x) \Psi(0, x) \, dx + \int_{\mathbb{R}^N} u_1(x) \Psi(0, x) \, dx \\ & = \int_{\Sigma_T} u \Psi_{tt} \, dx \, dt + \int_{\Sigma_T} u D_{t|T}^\alpha \Psi \, dx \, dt + \int_{\Sigma_T} u ((-\Delta)^{\frac{\beta}{2}} \Psi) \, dx \, dt. \end{aligned} \quad (3.9)$$

To estimate

$$\int_{\Sigma_T} u \Psi_{tt} \, dx \, dt,$$

we observe that

$$\int_{\Sigma_T} u \Psi_{tt} \, dx \, dt = \int_{\Sigma_T} u (h\Psi)^{\frac{1}{p}} \Psi_{tt} (h\Psi)^{-\frac{1}{p}} \, dx \, dt,$$

we have also

$$\int_{\Sigma_T} u D_{t|T}^\alpha \Psi \, dx \, dt = \int_{\Sigma_T} u (h\Psi)^{\frac{1}{p}} D_{t|T}^\alpha \Psi (h\Psi)^{\frac{-1}{p}} \, dx \, dt,$$

and

$$\int_{\Sigma_T} u ((-\Delta)^{\frac{\beta}{2}} \Psi) \, dx \, dt = \int_{\Sigma_T} u (h\Psi)^{\frac{1}{p}} ((-\Delta)^{\frac{\beta}{2}} \Psi) (h\Psi)^{\frac{-1}{p}} \, dx \, dt.$$

An application of the following ϵ -Young's inequality

$$ab \leq \epsilon a^p + C(\epsilon) b^q, \quad a > 0, \quad b > 0, \quad \epsilon > 0, \quad pq = p + q \text{ and } C(\epsilon) = (\epsilon p)^{\frac{-q}{p}} q^{-1},$$

to the first integral of the right hand side of (3.9), we obtain

$$\int_{\Sigma_T} u \Psi_{tt} \, dx \, dt \leq \epsilon \int_{\Sigma_T} |u|^p h\Psi \, dx \, dt + C(\epsilon) \int_{\Sigma_T} |\Psi_{tt}|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt,$$

and for the second integral of the right hand side of (3.9), we get

$$\int_{\Sigma_T} u D_{t|T}^\alpha \Psi \, dx \, dt \leq \epsilon \int_{\Sigma_T} |u|^p h\Psi \, dx \, dt + C(\epsilon) \int_{\Sigma_T} \left(|D_{t|T}^\alpha \Psi| \right)^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt.$$

Similarly for the third integral of the right hand side of (3.9), we have

$$\left| \int_{\Sigma_T} u (-\Delta)^{\frac{\beta}{2}} \Psi \, dx \, dt \right| \leq \epsilon \int_{\Sigma_T} |u|^p h\Psi \, dx \, dt + C(\epsilon) \int_{\Sigma_T} \left| (-\Delta)^{\frac{\beta}{2}} (\Psi) \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt.$$

Finally, we get

$$\begin{aligned} \int_{\Sigma_T} |u|^p h\Psi \, dx \, dt &\leq C \int_{\Sigma_T} |\Psi_{tt}|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt + C \int_{\Sigma_T} \left| D_{t|T}^\alpha \Psi \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt \\ &\quad + C \int_{\Sigma_T} \left| (-\Delta)^{\frac{\beta}{2}} (\Psi) \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt. \end{aligned} \tag{3.10}$$

By the choice of Ψ , it is easy to show that

$$\left\{ \begin{array}{l} \int_{\Sigma_T} |\Psi_{tt}|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt < \infty, \quad \int_{\Sigma_T} \left| D_{t|T}^\alpha \Psi \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt < \infty, \\ \int_{\Sigma_T} \left| (-\Delta)^{\frac{\beta}{2}} (\Psi) \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, dx \, dt < \infty. \end{array} \right.$$

At this stage, we introduce the scaled variables:

$$\tau = tR^{-1}, \quad \zeta = xR^{-\frac{\alpha}{\beta}}.$$

Using the fact that

$$dxdt = R^{\frac{N\alpha}{\beta}+1} d\zeta d\tau, \quad \Psi_t = R^{-1} \Psi_\tau, \quad \Psi_{tt} = R^{-2} \Psi_{\tau\tau}, \quad (-\Delta)_x^{\frac{\beta}{2}} \Psi = R^{-\alpha} (-\Delta)_\zeta^{\frac{\beta}{2}} \Psi, \quad D_{t|T}^\alpha \Psi = R^{-\alpha} D_{\tau|RT}^\alpha \Psi,$$

and setting

$$\Omega = \left\{ (\tau, \zeta) \in \mathbb{R}^+ \times \mathbb{R}^N; 1 \leq \tau^2 + |\zeta|^{\frac{2\beta}{\alpha}} \leq 2 \right\}, \quad \varphi(\tau, \zeta) = \tau^2 + |\zeta|^{\frac{2\beta}{\alpha}},$$

we arrive at

$$\begin{aligned} \int_{\Sigma_T} |u|^p h \Psi \, dx \, dt &\leq CR^{\theta_1} \int_{\Omega} |(\Psi_{\tau\tau})(\varphi)|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, d\zeta \, d\tau \\ &\quad + CR^{\theta_2} \int_{\Omega} \left| (D_{\tau|RT}^{\alpha})(\varphi) \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, d\zeta \, d\tau \\ &\quad + CR^{\theta_3} \int_{\Omega} \left| (-\Delta)^{\frac{\beta}{2}} \Psi(\varphi) \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, d\zeta \, d\tau. \end{aligned} \quad (3.11)$$

Where

$$\left\{ \begin{array}{l} \theta_1 = \frac{N\alpha}{\beta} + 1 - \frac{2p}{p-1} - \frac{1}{p-1} \left(\frac{\alpha}{\beta} \mu + \nu \right), \\ \theta_2 = \frac{N\alpha}{\beta} + 1 - \frac{\alpha p}{p-1} - \frac{1}{p-1} \left(\frac{\alpha}{\beta} \mu + \nu \right), \\ \theta_3 = \frac{N\alpha}{\beta} + 1 - \frac{\alpha p}{p-1} - \frac{1}{p-1} \left(\frac{\alpha}{\beta} \mu + \nu \right). \end{array} \right.$$

One can easily observe that: $\theta_1 < \theta_2 = \theta_3$, we infer that

$$\begin{aligned} \int_{\Sigma_T} |u|^p h \Psi \, dx \, dt &\leq CR^{\theta} \left[\int_{\Omega} |(\Psi_{\tau\tau})(\varphi)|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, d\zeta \, d\tau \right. \\ &\quad + \int_{\Omega} \left| (D_{\tau|RT}^{\alpha})(\varphi) \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, d\zeta \, d\tau \\ &\quad \left. + \int_{\Omega} \left| (-\Delta)^{\frac{\alpha}{2}} \Psi(\varphi) \right|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} \, d\zeta \, d\tau \right], \end{aligned} \quad (3.12)$$

where $R > 0$, large and

$$\theta = \theta_2 = \frac{1}{\beta(p-1)} \left\{ [\alpha N + \beta(1-\alpha)]p - \alpha(N + \mu) - \beta(1 + \nu) \right\}.$$

It is clear that $\alpha N + \beta(1-\alpha) > 0$, thus, we distinguish two cases:

- If

$$\theta < 0 \Leftrightarrow p < \frac{\alpha(N + \mu) + \beta(1 + \nu)}{\alpha N + \beta(1 - \alpha)},$$

then the right-hand side of (3.12) goes to 0 when R tends to infinity, we pass to the limit in the left hand side, as R goes to $+\infty$, we get

$$\lim_{R \rightarrow +\infty} \int_{\Sigma_T} h |u|^p \Psi \, dx \, dt = 0.$$

Using the Lebesgue dominated convergence theorem, the continuity in time and space of u and the fact that $\Psi(t, x) \rightarrow 1$ as $R \rightarrow +\infty$, we infer that

$$\int_{\mathbb{R}^+ \times \mathbb{R}^N} h |u|^p dx dt = 0.$$

Therefore, if u exists then necessarily $u \equiv 0$ *a. e.* on $\mathbb{R}^+ \times \mathbb{R}^N$.

• If

$$\theta = 0 \Leftrightarrow p = \frac{\alpha(N + \mu) + \beta(1 + \nu)}{\alpha N + \beta(1 - \alpha)},$$

then we have

$$\int_{\mathbb{R}^+ \times \mathbb{R}^N} |u|^p h dx dt < +\infty. \quad (3.13)$$

By using (3.9) we obtain

$$\begin{aligned} & \int_{\Sigma_T} h |u|^p \Psi dx dt + \int_{\mathbb{R}^N} u_0(x) \Psi(0, x) dx + \int_{\mathbb{R}^N} u_1(x) \Psi(0, x) dx \\ & \leq \int_{\Sigma_T} u (h\Psi)^{\frac{1}{p}} |\Psi_{tt}| (h\Psi)^{\frac{-1}{p}} dx dt + \int_{\Sigma_T} u (h\Psi)^{\frac{1}{p}} \left| D_{t|T}^\alpha \right| (h\Psi)^{\frac{-1}{p}} dx dt \\ & + \int_{\Sigma_T} u (h\Psi)^{\frac{1}{p}} \left| (-\Delta)^{\frac{\beta}{2}} \Psi \right| (h\Psi)^{\frac{-1}{p}} dx dt. \end{aligned} \quad (3.14)$$

Accordingly, using Hölder's inequality in the right hand side of (3.14), yields

$$\begin{aligned} & \int_{\Sigma_T} h |u|^p \Psi dx dt \leq \left(\int_{\Sigma_T} u^p h \Psi dx dt \right)^{\frac{1}{p}} \left(\int_{\Sigma_T} |\Psi_{tt}|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} dx dt \right)^{\frac{p-1}{p}} \\ & + \left(\int_{\Sigma_T} u^p h \Psi dx dt \right)^{\frac{1}{p}} \left(\int_{\Sigma_T} \left(\left| D_{t|T}^\alpha \right| \right)^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} dx dt \right)^{\frac{p-1}{p}} \\ & + \left(\int_{\Sigma_T} u^p h \Psi dx dt \right)^{\frac{1}{p}} \left(\int_{\Sigma_T} \left(\left| (-\Delta)^{\frac{\beta}{2}} \Psi \right| \right)^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} dx dt \right)^{\frac{p-1}{p}}. \end{aligned}$$

We easily see that

$$\begin{aligned} & \int_{\Sigma_T} h |u|^p \Psi dx dt \leq \left(\int_{\Sigma_T} u^p h \Psi dx dt \right)^{\frac{1}{p}} \times \left[\left(\int_{\Sigma_T} |\Psi_{tt}|^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} dx dt \right)^{\frac{p-1}{p}} \right. \\ & + \left(\int_{\Sigma_T} \left(\left| D_{t|T}^\alpha \right| \right)^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} dx dt \right)^{\frac{p-1}{p}} \\ & \left. + \left(\int_{\Sigma_T} \left(\left| (-\Delta)^{\frac{\beta}{2}} \Psi \right| \right)^{\frac{p}{p-1}} (h\Psi)^{\frac{-1}{p-1}} dx dt \right)^{\frac{p-1}{p}} \right]. \end{aligned}$$

Because $\theta = 0$, we get from (3.13) that

$$\begin{aligned} \int_{\Sigma_T} h |u|^p \Psi \, dx \, dt &\leq \left(\int_{\Omega_2} u^p h \Psi \, dx \, dt \right)^{\frac{1}{p}} \times \left[\left(\int_{\Omega_1} |\Psi_{\tau\tau}(\varphi)|^{\frac{p}{p-1}} (h\Psi(\varphi))^{\frac{-1}{p-1}} \, d\zeta \, d\tau \right)^{\frac{p-1}{p}} \right. \\ &\quad + \left(\int_{\Omega_1} (|D_{\tau|RT}^{\alpha}(\varphi)|)^{\frac{p}{p-1}} (h\Psi(\varphi))^{\frac{-1}{p-1}} \, d\zeta \, d\tau \right)^{\frac{p-1}{p}} \\ &\quad \left. + \left(\int_{\Omega_1} (|(-\Delta)^{\frac{\beta}{2}} \Psi(\varphi)|)^{\frac{p}{p-1}} (h\Psi(\varphi))^{\frac{-1}{p-1}} \, d\zeta \, d\tau \right)^{\frac{p-1}{p}} \right], \end{aligned}$$

where

$$\Omega_1 = \left\{ (\tau, \zeta) \in \mathbb{R}^+ \times \mathbb{R}^N; 1 \leq \tau^2 + |\zeta|^{\frac{2\beta}{\alpha}} \leq 2 \right\},$$

and

$$\Omega_2 = \left\{ (t, x) \in \mathbb{R}^+ \times \mathbb{R}^N; R^2 \leq t^2 + |x|^{\frac{2\beta}{\alpha}} \leq 2R^2 \right\}.$$

Taking into account the fact that $\int_{\mathbb{R}^+ \times \mathbb{R}^N} |u|^p h \, dx \, dt < +\infty$, we obtain

$$\lim_{R \rightarrow +\infty} \int_{\Omega_2} |u|^p h \Psi \, dx \, dt = 0,$$

hence, we conclude that

$$\int_{\mathbb{R}^+ \times \mathbb{R}^N} |u|^p h \, dx \, dt = 0.$$

Whereupon $u \equiv 0$. We deduce that no nontrivial global solution is possible. This finishes the proof.

Remark 3.1. We observe that in the case $\alpha = 1$, $\beta = 2$, $\mu = \nu = 0$, we retrieve the Fujita's critical exponent $p_c = 1 + \frac{2}{N}$.

REFERENCES

- [1] M. BERBICHE, A. HAKEM. Necessary conditions for the existence and sufficient conditions for the nonexistence of solutions to a certain fractional telegraph equation, *Memoirs on Differential Equations and Mathematical physics* 2012; 56: 37-55.
- [2] A. Z. FINO. Critical exponent for damped wave equations with nonlinear memory, *Nonlinear Analysis* 2011;(74): 5495-5505.
- [3] A. Z. FINO, H. IBRAHIM A. WEHBE. A blow-up result for a nonlinear damped wave equation in exterior domain: The critical case, *Computers Mathematics with Applications* 2017; 73(11): 2415-2420.
- [4] H. FUJITA. On the blowing up of solutions of the problem for $u_t = \Delta u + u^{1+\alpha}$, *J. Fac. Sci.Univ. Tokyo* 1966;(13): 109 - 124.
- [5] M. GUEDDA, M. KIRANE. A note on nonexistence of global solutions to a nonlinear integral equation, *Bull. Belg. Math. Soc. Simon Stevin* 1999; (6): 491 - 497.

- [6] M. GUEDDA, M. KIRANE. Local and global nonexistence of solutions to semilinear evolution equations, *Electronic Journal of Differential Equations* 2002; (09): 149-160.
- [7] B. GUO, X. PU, F. HUANG. *Fractional Partial Differential Equations and Their Numerical Solutions*, World Scientific Publishing Co. Pte. Ltd. Beijing, China 2011.
- [8] A. HAKEM. Nonexistence of weak solutions for evolution problems on \mathbb{R}^N , *Bull. Belg. Math. Soc* 2005;(12): 73-82.
- [9] A. HAKEM, M. BERBICHE. On the blow-up behaviour of solutions to semi-linear wave models with fractional damping, *IAENG International Journal of Applied Mathematics* 2011;(41:3): 206-212.
- [10] R. IKEHATA. Small data global existence of solutions for dissipative wave equations in an exterior domain, *Funkcial. Ekvac* 2002;(44): 259-269.
- [11] M. KIRANE, Y. LASKRI N.-E.TATAR. Critical exponents of fujita type for certain evolution equations and systems with spation-temporal fractional derivatives, *J. Math. Anal. Appl* 2005;(312): 488-501.
- [12] W. MINGXIN. Global existence and finite time blow up for a reaction-diffusion system, *Z. Angew. Math. Phys* 2000;(51): 160-167.
- [13] T. OGAWA, H. TAKIDA. Non-existence of weak solutions to nonlinear damped wave equations in exterior domains, *J. Nonlinear analysis* 2009;(70): 3696-3701.
- [14] I. PODLUBNY. *Fractional differential equations*, Mathematics in Science and Engineering, vol 198, Academic Press, New York, 1999.
- [15] S.I. POHOZAEV, A. TESEI. Nonexistence of Local Solutions to Semilinear Partial Differential Inequalities, *Nota Scientifica* 01/28, Dip. Mat. Università "La Sapienza", Roma 2001.
- [16] S. POHOZAEV, L. VERON. Blow up results for nonlinear hyperbolic enequalities, *Ann. Scuola Norm. Sup. Pisa Cl. Sci* 2000; (XXIX)(4): 393-420.
- [17] C. POZRIKIDIS. *The fractional Laplacian*, Taylor Francis Group, LLC /CRC Press, Boca Raton (USA), 2016.
- [18] S. G. SAMKO, A. A. KILBAS, O. I. MARICHEV. *Fractional integrals and derivatives: Theory and applications*, Gordan and Breach Sci. Publishers, Yverdon, 1993.
- [19] G.TODOROVA, B.YORDANOV. Critical Exponent for a Nonlinear Wave Equation with Damping, *Journal of Differential Equations* 2001;(174): 464-489.
- [20] Q. S. ZHANG. A blow up result for a nonlinear wave equation with damping: the critical case, *C. R. Acad.Sci. paris* 2001; (333)(2): 109-114.

LABORATORY ACEDP, DJILLALI LIABES UNIVERSITY, 22000 SIDI BEL ABBES, ALGERIA.

Email address: djilalimedjahed@yahoo.fr

LABORATORY ACEDP, DJILLALI LIABES UNIVERSITY, 22000 SIDI BEL ABBES, ALGERIA.

Email address: hakemali@yahoo.com