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EXTENDED TRAVELING WAVE SOLUTIONS FOR SOME INTEGRO PARTIAL DIFFERENTIAL EQUATIONS

SERIFE MUGE EGE

Abstract. In this study, an extended method is implemented to find traveling wave solutions of two integro partial differential equations. The exact particular solutions containing hyperbolic function type are obtained. By using symbolic computation it is shown that this method is efficient mathematical tool for solving problems in nonlinear science.

1. Introduction

Many nonlinear physical pheonemena such as liquid dynamics, elasticity, chemical kinematics, relativity, optical fiber etc. are modelled by nonlinear partial differential equations. Therefore traveling wave solutions of nonlinear partial differential equations have importance in real world problems. Due to these solutions give information about the character of physical events, it is required to powerful methods such as the auxiliary equation method [1], extended auxiliary equation method [2], Painleve method [3], inverse scattering method [4], simple equation method [5], modified simple equation method [6, 7, 8, 9], extended simple equation method [10], G'/G expansion method [11, 12, 13], $\tan(\phi(\xi)/2)-$ expansion method [14], tanh method [15], extended tanh method [16], $\exp(-phi(xi))$-expansion method [17], subequation method [18],modified Kudryashov method [19], generalized Kudryashov method

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[20, 21, 26], extended Kudryashov method [22], ansatz method [23] and so on.

In this paper, by inspiring the modified Kudryashov method, an extended method is executed to find the traveling wave solutions for two integro partial differential equations, namely, $(1+1)$ - dimensional and $(2+1)$ - dimensional Ito's equations given as [24, 25, 26]:

$$v_{tt} + v_{xxxt} + 3(v_x u_t + v v_{xt}) + 3v_{xx} \int_{-\infty}^{x} v_t dx = 0$$

where v is the function of (x,t) and

$$v_{tt} + v_{xxxt} + 3(2v_x v_t + v v_{xt}) + 3v_{xx} \int_{-\infty}^{x} v_t dx + \alpha v_{yt} + \beta v_{xt} = 0 \tag{1.1}$$

where v is the function of (x,y,t).

The remnant of this paper organized as follows: In the following section we have a brief review on the extended method. In section 3, we use this method to get traveling wave solutions of Ito's equations. Finally, conclusions are given in Section 4.

2. Methodology

The extended method is described systematically in this section [22].

Step 1. We suppose that given nonlinear partial differential equation for $u(x,t)$ to be in the form:

$$P\left(u, u_t, u_x, u_y, u_z, u_{xy}, u_{yz}, u_{xz}, ...\right) = 0 \tag{2.2}$$

which can be reduced to an ordinary differential equation. Then Eq.(2.2) reduces to a nonlinear ordinary differential equation of the form:

$$H(u, u_\mu, u_{\mu\mu}, u_{\mu\mu\mu}, \ldots) = 0 \tag{2.3}$$

under the wave transformation

$$u(x,y,z,\ldots,t) = u(\mu), \quad \mu = k(x+ct) \quad \texttt{or} \quad \mu = x - ct, \tag{2.4}$$

where k and c are constants.

Step 2. Suppose that the traveling wave solutions of Eq.(2.3) to be as follows:

$$u(\mu) = \sum_{i=0}^{N} a_i Z^i(\mu) \tag{2.5}$$

where $a_i (i = 0, 1, 2, .., N)$ are constants such that $a_N \neq 0$ and $Z = \pm\frac{1}{\sqrt{1 \pm a^{2\mu}}}$. The function Z is the solution of equation of the auxilary ordinary differential equation

$$Z_\mu = lna(Z^3 - Z). \tag{2.6}$$

Step 3. In order to calculate the positive integer N in formula (2.5) we consider the homogenous balance between the highest order nonlinear terms and highest order derivatives in Eq. (2.3). Supposing $u^s(\mu)u^{(l)}(\mu)$ and $(u^{(r)}(\mu))^p$ are the highest order nonlinear terms of Eq. (2.3) and we have

$$N = \frac{2(l-pr)}{p-s-1}. \tag{2.7}$$

Step 4. Substituting Eq.(2.5) into Eq.(2.3) and equating the coefficients of Z^i to zero, we obtain a system of algebraic equations. By solving this system with the help of Mathematica packet program, we get the traveling wave solutions of Eq.(2.3).

3. Applications

3.1. **(1+1) dimensional integro-differential Ito Equation.** We first apply the method to $(1+1)$ - dimensional integro-differential Ito equation in the form:

$$v_{tt} + v_{xxxt} + 3(v_x u_t + v v_{xt}) + 3v_{xx}\int_{-\infty}^{x} v_t dx = 0 \tag{3.8}$$

where v is the function of (x,t).

We use the transformation

$$v(x,t) = u_x(x,t).$$

This transformation carries Eq.(3.8) into following differential equation:

$$u_{xtt} + u_{xxxxt} + 3(u_{xx}u_{xt} + u_x u_{xxt}) + 3u_{xxx}u_t = 0. \tag{3.9}$$

Then, using travelig wave transformation (2.4) we have

$$-cu''' + u^{(v)} - 3c(u''u'' + u'u'') - 3cu'''u' = 0. \tag{3.10}$$

where $' = \frac{d}{d\xi}$. By integrating Eq.(3.10), we obtain, upon setting the integration constant to zero,

$$u''' + 3c(u')^2 - cu' = 0. \tag{3.11}$$

Then using the transformation $\omega = u'$ Eq.(3.11) can be written as

$$\omega'' + 3\omega^2 - c\omega = 0. \tag{3.12}$$

Also we take

$$\omega(\mu) = \sum_{i=0}^{N} a_i Z^i \tag{3.13}$$

where $Z(\mu) = \pm\frac{1}{(1\pm e^{2\mu})^{1/2}}$. We note that the function Z is the solution of $Z_\mu = Z^3 - Z$. Balancing the highest order derivative and nonlinear term we calculate

$$N = 4. \tag{3.14}$$

Thus, we have

$$\omega(\mu) = a_0 + a_1 Z(\mu) + a_2 Z^2(\mu) + a_3 Z^3(\mu) + a_4 Z^4(\mu) \tag{3.15}$$

and substituting derivatives of $\omega(\mu)$ with respect to μ in Eq.(3.15). The required derivatives in Eq. (3.12) are obtained

$$\omega_\mu = (Z^3 - Z)(a_1 + 2a_2 Z + 3a_3 Z^2 + 4a_4 Z^3), \tag{3.16}$$

$$\omega_{\mu\mu} = (Z^3 - Z)[24a_4 Z^5 + 15a_3 Z^4 + (8a_2 - 16a_4)Z^3 \tag{3.17}$$

$$+ (3a_1 - 9a_3)Z^2 - 4a_2 - a_1]. \tag{3.18}$$

Substituting derivatives Eq.(3.15) and Eq.(3.16) into Eq.(3.12) and accumulate the coefficient of each power of Z^i, setting each of coefficient to zero, solving the resulting system of algebraic equations we get the following solutions:

Case 1:

$$a_0 = -\frac{4}{3}, \quad a_1 = a_1, \quad a_2 = 8, \quad a_3 = a_3, \tag{3.19}$$

$$a_4 = -8, \quad c = -4. \tag{3.20}$$

Inserting Eq.(3.19) into Eq.(3.15), we obtain the following solutions of Eq.(3.8) with respect to traveling wave transformation (2.4)

$$v_1(\mu) = -\frac{4}{3} + \frac{a_1(1 + e^{2\mu}) + a_3}{(1 + e^{2\mu})^{3/2}} + \frac{2}{cosh^2(\mu)}, \tag{3.21}$$

$$v_2(\mu) = -\frac{4}{3} + \frac{a_1(1 - e^{2\mu}) + a_3}{(1 - e^{2\mu})^{3/2}} - \frac{2}{sinh^2(\mu)}. \tag{3.22}$$

Thus, we obtain new exact solutions to Eq.(3.8)

$$v_1(x,t) = -\frac{4}{3} + \frac{a_1(1 + e^{2x+8t}) + a_3}{(1 + e^{2x+8t})^{3/2}} + \frac{2}{cosh^2(x+4t)}, \tag{3.23}$$

$$v_2(x,t) = -\frac{4}{3} + \frac{a_1(1 - e^{2x+8t}) + a_3}{(1 - e^{2x+8t})^{3/2}} - \frac{2}{sinh^2(x+4t)}. \tag{3.24}$$

Case 2:

$$a_0 = 0, \quad a_1 = a_1, \quad a_2 = -8, \quad a_3 = a_3, \tag{3.25}$$

$$a_4 = -8, \quad c = 4 \tag{3.26}$$

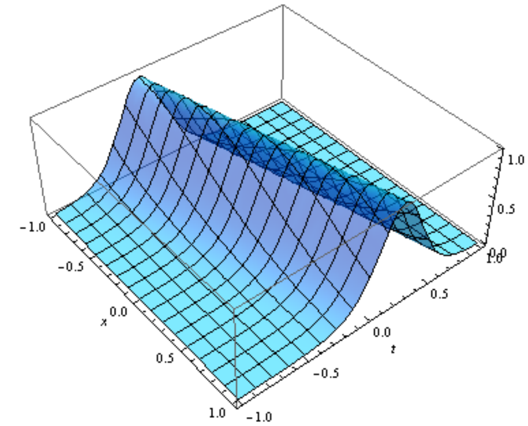

FIGURE 1. The exact solution v_1 of Eq.(3.8)

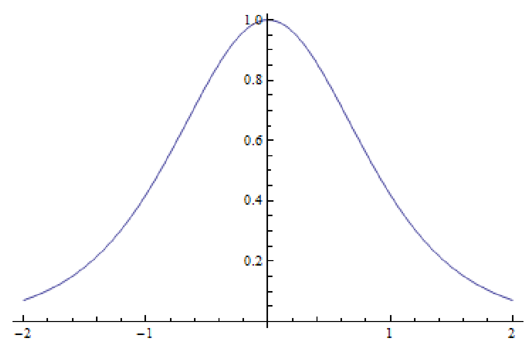

FIGURE 2. The projection of v_1 at $t = 0$

Inserting Eq.(3.25) into Eq.(3.15), we obtain the following solutions of Eq.(3.8) with respect to traveling wave transformation (2.4)

$$v_3(\mu) = \frac{a_1(1+e^{2\mu}) + a_3}{(1+e^{2\mu})^{3/2}} + \frac{2}{cosh^2(\mu)}, \tag{3.27}$$

$$v_4(\mu) = \frac{a_1(1-e^{2\mu}) + a_3}{(1-e^{2\mu})^{3/2}} - \frac{2}{sinh^2(\mu)}. \tag{3.28}$$

Thus, we get new exact solutions to Eq.(3.8)

$$u_3(x,t) = \frac{a_1(1+e^{2x-8t}) + a_3}{(1+e^{2x-8t})^{3/2}} + \frac{2}{cosh^2(x-4t)}, \tag{3.29}$$

$$u_4(x,t) = \frac{a_1(1-e^{2x-8t}) + a_3}{(1-e^{2x-8t})^{3/2}} - \frac{2}{sinh^2(x-4t)}. \tag{3.30}$$

3.2. **(2+1) dimensional integro-differential Ito Equation.** We secondly apply the method to $(2+1)$ - dimensional integro-differential Ito equation in the form:

$$v_{tt} + v_{xxxt} + 3(2v_xv_t + vv_{xt}) + 3v_{xx}\int_{-\infty}^{x} v_t dx + \alpha v_{yt} + \beta v_{xt} = 0 \tag{3.31}$$

where v is the function of (x, y, t).

Using the transformation

$$v(x,t) = u_x(x,t)$$

Eq.(3.31) turns into following differential equation:

$$u_{xtt} + u_{xxxxt} + 3(2u_{xx}u_{xt} + u_x u_{xxt}) + 3u_{xxx}u_t + \alpha u_{xyt} + \beta u_{xxt} = 0. \tag{3.32}$$

By considering the traveling wave transformation $\mu = x + y - ct$, Eq.(3.33) can be reduced to the following ordinary differential equation:

$$(c - \alpha - \beta)u''' - u^{(v)} - 3((u')^2)'' = 0 \tag{3.33}$$

where $' = \frac{d}{d\xi}$. If we integrate twice, we get

$$(c - \alpha - \beta)u' - u''' - 3(v')^2 = 0. \tag{3.34}$$

Then using the transformation $\omega = u'$ Eq.(3.34) can be written as

$$(c - \alpha - \beta)\omega - \omega'' - 3\omega^2 = 0. \tag{3.35}$$

Also we take

$$\omega(\mu) = \sum_{i=0}^{N} a_i Z^i \tag{3.36}$$

where $Z(\mu) = \pm\frac{1}{(1 \pm e^{2\mu})^{1/2}}$. We note that the function Z is the solution of $Z_\mu = Z^3 - Z$. Balancing the highest order derivative and nonlinear term we calculate

$$N = 4. \tag{3.37}$$

Thus, we have

$$\omega(\mu) = a_0 + a_1 Z(\mu) + a_2 Z^2(\mu) + a_3 Z^3(\mu) + a_4 Z^4(\mu) \tag{3.38}$$

and substituting derivatives of $\omega(\mu)$ with respect to μ in Eq.(3.38). The required derivatives in Eq. (3.35) are obtained

$$\omega_\mu = (Z^3 - Z)(a_1 + 2a_2 Z + 3a_3 Z^2 + 4a_4 Z^3), \tag{3.39}$$

$$\begin{aligned} \omega_{\mu\mu} &= (Z^3 - Z)[24a_4 Z^5 + 15a_3 Z^4 + (8a_2 - 16a_4)Z^3 \\ &+ (3a_1 - 9a_3)Z^2 - 4a_2 - a_1]. \end{aligned} \tag{3.40}$$

Substituting derivatives Eq.(3.38) and Eq.(3.39) into Eq.(3.35) and accumulating the coefficient of each power of Z^i, setting each of coefficient to zero, solving the resulting algebraic equation system we obtain the following solutions:

Case 1:

$$a_0 = -\frac{4}{3}, \quad a_1 = 0, \quad a_2 = 8, \quad a_3 = 0, \tag{3.41}$$

$$a_4 = -8, \quad c = -4 + \alpha + \beta \tag{3.42}$$

Inserting Eq.(3.41) into Eq.(3.38), we obtain the following solutions of Eq.(3.31) with respect to traveling wave transformation $\mu = x + y - ct$

$$v_1(\mu) = -\frac{4}{3} + \frac{2}{cosh^2(\mu)}, \tag{3.43}$$

$$v_2(\mu) = -\frac{4}{3} - \frac{2}{sinh^2(\mu)}. \tag{3.44}$$

Thus, we obtain new exact solutions to Eq.(3.31) in the form:

$$v_1(x, y, t) = -\frac{4}{3} + \frac{2}{cosh^2(x + y - (4 - \alpha - \beta)t)}, \tag{3.45}$$

$$v_2(x, t) = -\frac{4}{3} - \frac{2}{sinh^2(x + y - (4 - \alpha - \beta)t)}. \tag{3.46}$$

Case 2:

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = 8, \quad a_3 = 0, \tag{3.47}$$

$$a_4 = -8, \quad c = 4 + \alpha + \beta \tag{3.48}$$

Inserting Eq.(3.47) into Eq.(3.38), we get the following solutions of Eq(3.31) with respect to traveling wave transformation $\mu = x + y - ct$:

$$v_1(\mu) = \frac{2}{cosh^2(\mu)}, \tag{3.49}$$

$$v_2(\mu) = \frac{2}{sinh^2(\mu)}. \tag{3.50}$$

Thus, we obtain new exact solutions to Eq.(3.31) in the form:

$$v_3(x, t) = \frac{2}{cosh^2(x + y - (4 + \alpha + \beta)t)}, \tag{3.51}$$

$$v_4(x, t) = \frac{2}{sinh^2(x + y - (4 + \alpha + \beta)t)}. \tag{3.52}$$

4. Conclusion

In this work, the extended method is executed to construct exact solutions of nonlinear integro partial differential equations with constant coefficients. By using the proposed method we have successfully obtained analytical solutions of $(1+1)$ - dimensional and $(2+1)$ - dimensional Ito equations. Besides the solutions in [26] , hyperbolic function type solutions are obtained. In addition, change in the parameters effects the speed of the wave. The obtained solutions may have importance for some special technological and physical events. It can be concluded that this method is standard, effective and also convenient for solving nonlinear integro partial differential equations.

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Department of Mathematics

Ege University

Bornova, Izmir 35100 Turkey

E-mail address: serife.muge.ege@ege.edu.tr