

**DOMINATION AND INDEPENDENCE NUMBERS OF EXACT
ZERO-DIVISOR GRAPH OF THE RING \mathbb{Z}_n**

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Abstract. This study examines the structural properties of exact zero-divisor graph associated with commutative rings possessing a non-zero identity, emphasizing rings that are not integral domains. By focusing on key graph-theoretic parameters, such as domination and independence numbers, we provide a detailed analysis of their behavior in specific rings, including \mathbb{Z}_{p^2} , \mathbb{Z}_{p^k} , and \mathbb{Z}_{pq} . These findings reveal significant relationships between these parameters and the fundamental algebraic properties of rings, enhancing the comprehension of the interaction between graph theory and ring theory.

Keywords: Exact zero-divisor graph, Domination number, Independence number.

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1. INTRODUCTION

Exploring algebraic structures through their corresponding graphs has been a dynamic field of mathematical research, providing valuable insights into the relationship between algebra and graph theory. The zero-divisor graph of a commutative ring, first introduced by Beck [4], and later refined by Anderson and Livingston [1], exemplifies this interplay by Anderson et al. in [2]. In their definition, the vertex set comprises the nonzero zero-divisors of the ring, with adjacency determined by the zero-product condition. This framework has catalyzed numerous investigations into properties of such graphs, including connectivity, chromatic number, and domination number.

Graph theory, as a field, has a rich history and has developed into a powerful tool for analyzing relationships in various mathematical structures. Foundational text by Haynes et al. [6] has laid the groundwork for understanding concepts such as domination, independence, connectivity, and coloring in graphs. These graph-theoretic concepts have profound implications when applied to algebraic structures like rings, providing a means to explore their structural intricacies through visual and quantitative analysis.

In recent years, the concept of exact zero-divisors, introduced by Henriques and Sega [5], has provided a nuanced perspective on zero-divisor graphs. An exact zero-divisor is defined

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via principal annihilators with symmetric properties, thereby enabling the construction of the exact zero-divisor graph $E\Gamma(R)$ for a commutative ring R . The graph $E\Gamma(R)$ differs from the traditional zero-divisor graph $\Gamma(R)$ in its focus on a refined subset of zero-divisors, leading to richer structural properties and new avenues for research. As in [7], Lalchandani introduced the exact zero-divisor graph, particularly for the ring \mathbb{Z}_n , highlighting key differences between $\Gamma(\mathbb{Z}_n)$ and $E\Gamma(\mathbb{Z}_n)$. Later developments, such as in [8], explored further extensions, including properties of exact zero-divisor graphs for reduced rings, providing foundational results. These studies highlight the intricate interplay between the algebraic properties of a ring and the graph-theoretic properties of its associated graphs, a theme central to this paper.

The ring \mathbb{Z}_n , comprising integers modulo n , is a foundational object in algebra, with its arithmetic properties influencing a wide range of mathematical and applied studies [3]. When n is not a prime power, \mathbb{Z}_n admits nontrivial zero-divisors, making it a fertile ground for exploring zero-divisor graphs. Visweswaran and Lalchandani in [9] have shown that $E\Gamma(\mathbb{Z}_n)$ offers distinct structural properties compared to $\Gamma(\mathbb{Z}_n)$, particularly in terms of connectivity and bipartite decompositions. These insights form the backdrop for examining fundamental graph-theoretic parameters, such as domination and independence, in $E\Gamma(\mathbb{Z}_n)$.

Domination and independence are fundamental concepts in graph theory, with applications spanning optimization, network design, and social dynamics. The domination number, denoted $\gamma(G)$, quantifies the minimum size of a set of vertices such that every vertex in the graph is either in the set or adjacent to a vertex in the set [6]. On the other hand, the independence number $\alpha(G)$ captures the maximum size of a subset of vertices such that no two vertices in the subset are adjacent [10]. Both these invariants reveal critical information about the structure and behavior of a graph. For $E\Gamma(\mathbb{Z}_n)$, understanding these parameters sheds light on the intricate relationships among exact zero-divisors of \mathbb{Z}_n .

This article aims to advance the understanding of domination and independence numbers in the exact zero-divisor graph of the ring \mathbb{Z}_n . By building on foundational results, we develop methods to compute these invariants for various classes of \mathbb{Z}_n . Theoretical results are illustrated with concrete examples to elucidate the differences between $E\Gamma(\mathbb{Z}_n)$ and $\Gamma(\mathbb{Z}_n)$.

The organization of the paper is as follows. In Section 2, we review the definitions and fundamental preliminaries of exact zero-divisors and their associated graphs. Section 3 presents methods to compute the domination number for $E\Gamma(\mathbb{Z}_n)$ under various conditions on n . Also, we explore the independence number of $E\Gamma(\mathbb{Z}_n)$ for different values of n . We conclude with discussions on the implications of our findings and potential avenues for future research.

By integrating graph-theoretic techniques with algebraic insights, this work aims to contribute to the broader understanding of the structural properties of graphs derived from rings. The results presented here have potential implications for both pure mathematics and applications involving modular arithmetic and network analysis. Throughout the article, all rings considered are commutative rings with non-zero identity.

2. PRELIMINARIES

In this section, we introduce relevant terminology along with illustrative examples for this paper. For further details, readers are encouraged to consult references [3] and [10]. We begin with the following key definitions:

Definition 2.1. A zero-divisor in a commutative ring R with identity is a nonzero element $a \in R$ such that there exists a nonzero $b \in R$ with $ab = 0$.

Definition 2.2. The zero-divisor graph $\Gamma(R)$ of a commutative ring R is the graph where the vertex set consists of all nonzero zero-divisors of R , and two vertices a, b are adjacent if and only if $ab = 0$.

Definition 2.3. [7] An exact zero-divisor $a \in R$ is a zero-divisor for which there exists a non zero element $b \in R$ such that the annihilator $Ann(a) = \{r \in R \mid ra = 0\}$ is a principal ideal generated by b , and $Ann(b)$ is the principal ideal generated by a .

Definition 2.4. [7] The exact zero-divisor graph $E\Gamma(R)$ of a commutative ring R is the graph where the vertex set consists of all nonzero exact zero-divisors of R , and two vertices a, b are adjacent if and only if $Ann(a) = bR$ and $Ann(b) = aR$.

Definition 2.5. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set, which is a subset $D \subseteq V(G)$ such that every vertex in G is either in D or adjacent to a vertex in D .

Definition 2.6. The independence number $\alpha(G)$ of a graph G is the maximum cardinality of an independent set, which is a subset $I \subseteq V(G)$ such that no two vertices in I are adjacent.

We now compute the domination and independence numbers of $E\Gamma(R)$ for the rings \mathbb{Z}_6 , \mathbb{Z}_{27} , and \mathbb{Z}_{49} . For each ring, the graphs are constructed using exact zero-divisors, and the results are summarized.

Example 2.1. Let $R = \mathbb{Z}_{2 \cdot 3} = \mathbb{Z}_6$. Then the nonzero exact zero-divisors in \mathbb{Z}_6 are found to be $\{2, 3, 4\}$. The exact zero-divisor graph $E\Gamma(\mathbb{Z}_6)$ is as in Figure 1.

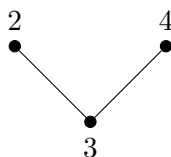
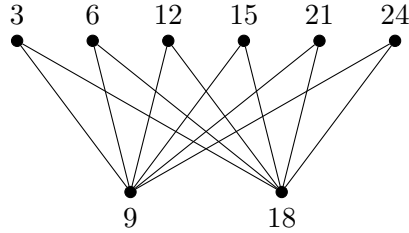


FIGURE 1. Exact Zero-Divisor Graph $E\Gamma(\mathbb{Z}_6)$

From the graph:

- **Domination Number:** $\gamma(E\Gamma(\mathbb{Z}_6)) = 1$, with $\{3\}$ as a dominating set.
- **Independence Number:** $\alpha(E\Gamma(\mathbb{Z}_6)) = 2$, with $\{2, 4\}$ as an independent set.

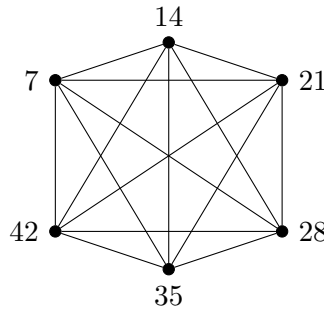
Example 2.2. Let $R = \mathbb{Z}_{27}$. Then the nonzero exact zero-divisors in \mathbb{Z}_{27} are $\{3, 6, 9, 12, 15, 18, 21, 24\}$. The exact zero-divisor graph $E\Gamma(\mathbb{Z}_{27})$ is in Figure 2.

FIGURE 2. Exact Zero-Divisor Graph $EG(\mathbb{Z}_{27})$

From the graph:

- **Domination Number:** $\gamma(EG(\mathbb{Z}_{27})) = 2$, with $\{9, 18\}$ as a dominating set.
- **Independence Number:** $\alpha(EG(\mathbb{Z}_{27})) = 6$, with $\{3, 6, 12, 15, 21, 24\}$ as an independent set.

Example 2.3. Let $R = \mathbb{Z}_{7^2} = \mathbb{Z}_{49}$. Then the nonzero exact zero-divisors in \mathbb{Z}_{49} are $\{7, 14, 21, 28, 35, 42\}$. The exact zero-divisor graph $EG(\mathbb{Z}_{49})$ is in Figure 3.

FIGURE 3. Exact Zero-Divisor Graph $EG(\mathbb{Z}_{49})$

From the graph:

- **Domination Number:** $\gamma(EG(\mathbb{Z}_{49})) = 1$, with $\{7\}$ as a dominating set.
- **Independence Number:** $\alpha(EG(\mathbb{Z}_{49})) = 1$, with $\{7\}$ as an independent set.

3. THE DOMINATION AND INDEPENDENCE NUMBERS OF $EG(\mathbb{Z}_n)$

Theorem 3.1. Let $R = \mathbb{Z}_{p^2}$, where p is prime number. Then the exact zero-divisor graph $EG(R)$ of the ring R is complete graph K_{p-1} with $p-1$ vertices and the domination number of $EG(R)$ is 1.

Proof. We know that the exact zero-divisor graph $EG(R)$ of the ring R is complete graph K_{p-1} with $p-1$ vertices for $R = \mathbb{Z}_{p^2}$ [8]. Consider any single vertex $v_1 \in V(EG(R))$, where $R = \mathbb{Z}_{p^2}$, since $EG(R)$ is complete graph with $p-1$ vertices, v_1 is adjacent to every other vertex in $V \setminus \{v_1\}$. Therefore, v_1 is adjacent to all the remaining $p-2$ vertices. Hence, the set $\{v_1\}$ is a dominating set because every vertex in $V \setminus \{v_1\}$ is adjacent to v_1 .

Thus, by the definition of the domination number, we conclude that

$$\gamma(EG(\mathbb{Z}_{p^2})) = 1.$$

□

Theorem 3.2. *Let $R = \mathbb{Z}_{pq}$, where p and q are distinct prime numbers such that $p < q$. Then the exact zero-divisor graph $E\Gamma(R)$ is isomorphic to the complete bipartite graph $K_{p-1,q-1}$, and its domination number is given by*

$$\gamma(E\Gamma(R)) = \begin{cases} 1, & \text{if } p = 2, \\ 2, & \text{if } p > 2. \end{cases}$$

Proof. The structure of $E\Gamma(\mathbb{Z}_{pq})$ aligns with the complete bipartite graph $K_{p-1,q-1}$, where the vertex sets correspond to the nonzero zero-divisors from the respective prime factors of pq [8]. When $p = 2$, the graph reduces to $K_{1,q-1}$, wherein the single vertex from the smaller partition is adjacent to all others. Hence, a single vertex suffices to dominate the graph, and we have:

$$\gamma(E\Gamma(\mathbb{Z}_{2q})) = 1.$$

For $p > 2$, each partite set has more than one vertex, and no single vertex can dominate both partitions. Selecting one vertex from each suffices to dominate the entire graph. Thus:

$$\gamma(E\Gamma(\mathbb{Z}_{pq})) = 2.$$

□

Theorem 3.3. *Let $R = \mathbb{Z}_{p^k}$, where p is a prime and $k \geq 3$ is odd. Then the domination number of the exact zero-divisor graph $E\Gamma(R)$ is*

$$\gamma(E\Gamma(\mathbb{Z}_{p^k})) = k - 1.$$

Proof. When $k \geq 3$ is odd, the graph $E\Gamma(\mathbb{Z}_{p^k})$ consists of $\frac{k-1}{2}$ disconnected components, each isomorphic to a complete bipartite graph [8]. These components arise from annihilating pairs (p^i, p^{k-i}) for $i = 1, 2, \dots, \frac{k-1}{2}$. In each component K_{m_i, n_i} , the partite sets have sizes $m_i = \varphi(p^i)$ and $n_i = \varphi(p^{k-i})$, both strictly greater than one. Thus, the domination number of each such bipartite component is exactly 2, as no single vertex can dominate both partitions. Since the components are disjoint, their domination numbers add. Therefore,

$$\gamma(E\Gamma(\mathbb{Z}_{p^k})) = 2 \cdot \frac{k-1}{2} = k - 1.$$

□

Theorem 3.4. *Let $R = \mathbb{Z}_{p^k}$, where p is a prime and $k \geq 4$ is even. Then the domination number of the exact zero-divisor graph $E\Gamma(R)$ is*

$$\gamma(E\Gamma(\mathbb{Z}_{p^k})) = k - 1.$$

Proof. For even $k \geq 4$, the graph $E\Gamma(\mathbb{Z}_{p^k})$ consists of $\frac{k}{2}$ disconnected components [8]. Among these, the first $\frac{k}{2} - 1$ components are complete bipartite graphs, each arising from annihilating pairs (p^i, p^{k-i}) , where $i = 1, 2, \dots, \frac{k}{2} - 1$. Each such component contributes 2 to the domination number. The remaining component, corresponding to $i = \frac{k}{2}$, involves zero-divisors with annihilators $\langle p^{k/2} \rangle$, forming a complete graph. In this case, a single vertex dominates the entire component. Combining all contributions, we obtain:

$$\gamma(E\Gamma(\mathbb{Z}_{p^k})) = 2 \cdot \left(\frac{k}{2} - 1 \right) + 1 = k - 1.$$

□

Theorem 3.5. *Let $R = \mathbb{Z}_{p^2}$, where p is a prime number, and let $E\Gamma(R)$ denote the exact zero-divisor graph of R . Then the independence number of $E\Gamma(R)$ is given by:*

$$\alpha(E\Gamma(\mathbb{Z}_{p^2})) = 1.$$

Proof. We want to show that the independence number of the exact zero-divisor graph $E\Gamma(R)$ for the ring $R = \mathbb{Z}_{p^2}$ is 1. To do this, we begin by understanding the structure of the graph $E\Gamma(R)$. It is known that $E\Gamma(\mathbb{Z}_{p^2})$ is a complete graph with $p - 1$ vertices [8].

In a complete graph, each vertex is connected to every other vertex. Consider any vertex v_1 in $E\Gamma(\mathbb{Z}_{p^2})$. Since the graph is complete, v_1 is adjacent to all other vertices. Therefore, the only independent set that can include v_1 is the set $\{v_1\}$, which contains just one vertex. This reasoning holds for any vertex in the graph. Hence, we conclude that the independence number of $E\Gamma(\mathbb{Z}_{p^2})$ is 1.

$$\alpha(E\Gamma(\mathbb{Z}_{p^2})) = 1$$

□

Theorem 3.6. *Let $R = \mathbb{Z}_{pq}$, where p and q are distinct prime numbers with $p < q$, and let $E\Gamma(R)$ denote the exact zero-divisor graph of R . Then the independence number of $E\Gamma(R)$ is given by:*

$$\alpha(E\Gamma(\mathbb{Z}_{pq})) = \phi(q) = q - 1.$$

Proof. To prove that the independence number of the exact zero-divisor graph $E\Gamma(R)$ for the ring $R = \mathbb{Z}_{pq}$, where p and q are distinct prime numbers, is $q - 1$, we start by considering the structure of the graph. The graph $E\Gamma(\mathbb{Z}_{pq})$ is a complete bipartite graph, specifically $K_{p-1, q-1}$ [8]. This means that the set of $p - 1$ vertices corresponds to the zero divisors in \mathbb{Z}_p , and the set of $q - 1$ vertices corresponds to the zero divisors in \mathbb{Z}_q . In this graph, every vertex in the first set is connected to every vertex in the second set, with no edges within either set.

Since $E\Gamma(\mathbb{Z}_{pq})$ is a complete bipartite graph, the largest independent set must come from one of the two sets, as there are no edges within either set. We can choose all the vertices from set B , which has $q - 1$ vertices. This is the larger of the two sets since $q > p$. Therefore, the largest independent set in $E\Gamma(\mathbb{Z}_{pq})$ contains $q - 1$ vertices.

Thus, we conclude that the independence number of $E\Gamma(\mathbb{Z}_{pq})$ is $q - 1$.

$$\alpha(E\Gamma(\mathbb{Z}_{pq})) = q - 1$$

□

Theorem 3.7. *Let $R = \mathbb{Z}_{p^k}$, where p is a prime number, and let $E\Gamma(\mathbb{Z}_{p^k})$, $k \geq 3$ denote the exact zero-divisor graph of R . Then the independence number of $E\Gamma(\mathbb{Z}_{p^k})$ is given by:*

$$\alpha(E\Gamma(\mathbb{Z}_{p^k})) = \begin{cases} \sum_{i=\frac{k+1}{2}}^{k-1} \phi(p^i), & \text{if } k \text{ is odd,} \\ \sum_{i=\frac{k}{2}+1}^{k-1} \phi(p^i) + 1, & \text{if } k \text{ is even.} \end{cases}$$

Proof. Let $R = \mathbb{Z}_{p^k}$, where $k \geq 3$. The vertices of $E\Gamma(\mathbb{Z}_{p^k})$ are all nonzero exact zero-divisors, given by $V = \{p^i \cdot m \mid m \in \mathbb{Z}_{p^k}^*, 1 \leq i \leq k - 1\}$, where $\mathbb{Z}_{p^k}^*$ consists of all elements in \mathbb{Z}_{p^k} (integers modulo p^k) that are coprime to p . By Theorem 3.2 of [8], $E\Gamma(\mathbb{Z}_{p^k})$ is a disjoint union of $\lfloor \frac{k}{2} \rfloor$ components.

For $i = 1, 2, \dots, \lfloor \frac{k}{2} \rfloor - 1$, each component corresponds to the annihilator pair $(\langle p^i \rangle, \langle p^{k-i} \rangle)$, forming a complete bipartite graph K_{X_i, Y_i} with $|X_i| = \phi(p^i)$ and $|Y_i| = \phi(p^{k-i})$. For $i = \frac{k}{2}$ (if k is even), the vertices form a complete graph $K_{\phi(p^{\frac{k}{2}})}$.

The independence number α of a graph is the size of its largest independent set. For K_{X_i, Y_i} , $\alpha(K_{X_i, Y_i}) = \max(|X_i|, |Y_i|) = \phi(p^{k-i})$, since $\phi(p^i) \leq \phi(p^{k-i})$. For $K_{\phi(p^{\frac{k}{2}})}$, $\alpha(K_{\phi(p^{\frac{k}{2}})}) = 1$.

Case 1: k is odd; When $k = 2m + 1$, $E\Gamma(\mathbb{Z}_{p^k})$ has m bipartite components. The independent set in each component corresponds to Y_i , where $i = \frac{k+1}{2}, \frac{k+3}{2}, \dots, k - 1$. Thus:

$$\alpha(E\Gamma(\mathbb{Z}_{p^k})) = \phi(p^{\frac{k+1}{2}}) + \phi(p^{\frac{k+3}{2}}) + \dots + \phi(p^{k-1}) = \sum_{i=\frac{k+1}{2}}^{k-1} \phi(p^i).$$

Case 2: k is even; When $k = 2m$, $E\Gamma(\mathbb{Z}_{p^k})$ has $m - 1$ bipartite components and one complete graph. The independent set in each bipartite component is Y_i for $i = \frac{k}{2} + 1, \frac{k}{2} + 2, \dots, k - 1$. The independent set in $K_{\phi(p^{\frac{k}{2}})}$ contributes 1. Therefore:

$$\alpha(E\Gamma(\mathbb{Z}_{p^k})) = \phi(p^{\frac{k}{2}+1}) + \phi(p^{\frac{k}{2}+2}) + \dots + \phi(p^{k-1}) + 1 = \sum_{i=\frac{k}{2}+1}^{k-1} \phi(p^i) + 1.$$

Therefore, the independence number of the exact zero-divisor graph $E\Gamma(\mathbb{Z}_{p^k})$ is given by:

$$\alpha(E\Gamma(\mathbb{Z}_{p^k})) = \begin{cases} \sum_{i=\frac{k+1}{2}}^{k-1} \phi(p^i), & \text{if } k \text{ is odd,} \\ \sum_{i=\frac{k}{2}+1}^{k-1} \phi(p^i) + 1, & \text{if } k \text{ is even.} \end{cases}$$

□

4. DOMINATION AND INDEPENDENCE NUMBERS IN $\Gamma(\mathbb{Z}_n)$ VS $E\Gamma(\mathbb{Z}_n)$

Table 4.1 presents a clear comparison of the domination and independence numbers of the zero-divisor graph $\Gamma(\mathbb{Z}_n)$ and the exact zero-divisor graph $E\Gamma(\mathbb{Z}_n)$ for selected values of n . This helps to highlight how these graph invariants vary between the two graph types.

n	$\gamma(\Gamma(\mathbb{Z}_n))$	$\gamma(E\Gamma(\mathbb{Z}_n))$	$\alpha(\Gamma(\mathbb{Z}_n))$	$\alpha(E\Gamma(\mathbb{Z}_n))$
16	1	3	4	5
27	1	2	6	6
32	1	4	14	12

TABLE 4.1. Comparison of domination and independence numbers for $\Gamma(\mathbb{Z}_n)$ and $E\Gamma(\mathbb{Z}_n)$

5. CONCLUSION

In this paper, we analyzed the properties of the exact zero-divisor graph $ET(R)$ of various commutative rings R of the form \mathbb{Z}_n , where n is a prime as well as composite number. This study explored the structure of exact zero-divisor graphs $ET(R)$ for rings of integers modulo n , focusing on their domination and independence numbers. The domination and independence numbers are tied to Euler's totient function, reflecting the deeper interplay between algebra and graph theory. These results reveal how the algebraic properties of rings influence the structure of their zero-divisor graphs, providing insights into both mathematical theory and practical applications in combinatorics and algebra. The domination and independence numbers computed for exact zero-divisor graphs are not only theoretically significant but also practically meaningful in systems where modular arithmetic and graph structure intersect. We highlight some applications below with examples and mathematical justification as following illustrations:

5.1. Optimal Monitoring in Ring-Structured Networks: In communication networks or fault-monitoring systems modeled by rings such as \mathbb{Z}_n , each exact zero-divisor can represent a node, and edges in $ET(\mathbb{Z}_n)$ capture mutual influence via annihilation. The domination number then corresponds to the smallest number of control points required to observe the entire system. As shown in Example 2.3, the graph $ET(\mathbb{Z}_{49})$ consists of six vertices with one dominating vertex namely 7. Since this vertex is adjacent to all others, selecting it ensures full coverage of the network. Thus, the domination number $\gamma(ET(\mathbb{Z}_{49})) = 1$ signifies that a single monitoring unit suffices. This principle applies to minimal sensor placement in systems with modular timing or synchronization constraints.

5.2. Construction of Non-Interfering Code-words in Modular Coding Schemes: In error-correcting code design, independent sets in graphs are often used to identify elements that behave distinctly under encoding operations. In $ET(\mathbb{Z}_n)$, an independent set corresponds to exact zero-divisors that do not annihilate each other, ensuring they operate without algebraic interference. From Example 2.2, the independent set $\{3, 6, 12, 15, 21, 24\}$ in $ET(\mathbb{Z}_{27})$ has size six, and no two elements within this set are adjacent. This makes them suitable for representing code-words with minimal overlap in structure. When mapped to vectors over \mathbb{Z}_{27} , these elements enable the design of modular codes with high Hamming distance and clear signal distinction which is essential in digital communication over noisy channels, especially where the underlying operations are defined over finite rings.

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