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STATISTICAL COMPACTNESS OF TOPOLOGICAL SPACES CONFINED BY WEIGHT FUNCTIONS

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ABSTRACT. A mapping of the form $\varrho : \mathbb{N} \longrightarrow [0, \infty)$ satisfying $\lim_{n\to\infty} \varrho(n) = \infty$ and $\lim_{n\to\infty} \frac{n}{\varrho(n)} \neq 0$ is called a weight function. By incorporating weight functions into the statistical framework, we come up with a new notion called weighted statistical compactness that extends the traditional notion of compactness. The paper involves studying the compactness properties via sequences and relationship between compactness variations. We also look into the nature of weighted statistical compactness with in sub-space and under open continuous onto maps. Weighted statistical compactness has also been given a finite intersection-like characterization.

Keywords: Countable compactness, s-compactness, Asymptotic density, Weighted density, Finite intersection property.

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1. INTRODUCTION

A group of points in a topological space are called dense when they are widely distributed throughout the space. The distances between the points are often used to calculate this density in a metric space. To determine the natural density (also known as asymptotic density) of a subset $A \subseteq \mathbb{N}$, one can measure how closely spaced out the points in A are in $\mathbb{N}$, $\mathbb{N}$ being all natural numbers set. It is described as

$$\delta(A) = \lim_{n\to\infty} \frac{1}{n} |\{k \leq n : k \in A \subseteq \mathbb{N}\}|.$$

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H. Fast [11], Schoenberg [13] expanded the sense of conventional convergence to statistical convergence by utilizing the concept of asymptotic density. In a space X, a sequence $\{z_n\}$ converges statistically to a point z if the natural density of the collection $\{n \in \mathbb{N} : z_n \in U\}$ (i.e., the part of the sequence's elements that fall within U) converges to 1 as n tends to infinity for every open set U containing z. i.e., $\delta(\{n \in \mathbb{N} : z_n \in U\}) = 1$ or equivalently, $\delta(\{n \in \mathbb{N} : z_n \notin U\}) = 0$ [12]. In 2012, Bhunia et al. [9] strengthened the idea that real sequences s-converge by using asymptotic density of order α, where $0 < \alpha < 1$. s^α-convergence restricts the notion of statistical convergence in topology. It introduces a parameter α that is important in characterizing the specific convergence behavior of sequences. Here, α represents a parameter influencing the convergence rate of sub-sequences, providing a more nuanced understanding of convergence than is achievable with traditional reasoning. A sequence $\{y_n\}$, s^α-converges to a point y in a space X if each open set U that contains y, produces

$$\delta^\alpha(\{n \in \mathbb{N} : x_n \notin U\}) = \lim_{n\to\infty} \frac{|\{n \in \mathbb{N} : x_n \notin U\}|}{n^\alpha} = 0.$$

Compactness and other covering features have been a very interesting topic for many mathematicians [4, 5, 6, 7] for a long period of time. Compactness in a topological space is a fundamental property that encapsulates what it means to be 'finite' in a general sense. Compact topological spaces are those that have a finite sub-cover for every open cover. Stated differently, regardless matter how we choose to cover the space, there is always a finite number of open sets that cover the entire space. Compactness has many implications and applications in the mathematical domains of analysis, geometry, and topology, to name a few. The notions of boundedness and finiteness are naturally extended from metric spaces to more general topological spaces. Other types of compactness, such as sequentially compact space, pseudo-compact space, and St-compact space, s^α-compact space [2, 3, 8] have been studied by many authors.

A mapping ϱ defined in the form $\varrho : \mathbb{N} \longrightarrow [0, \infty)$ such that $\lim_{n\to\infty} \varrho(n) = \infty$ and $\lim_{n\to\infty} \frac{n}{\varrho(n)} \neq 0$ is called a weight function [1]. For example, $h : \mathbb{N} \longrightarrow [0, \infty)$ such that $h(n) = n^\alpha$, where $0 < \alpha \leq 1$, $\varrho : \mathbb{N} \longrightarrow [0, \infty)$ such that $\varrho(n) = log(1 + n)$ are weight functions. Adem et al. [1] studied the concept of weighted convergence for real sequences. A sequence $\{y_n\}$ is said to s_ϱ-converge to the point y in a space X if each neighborhood U of y produces

$$\delta_\varrho(\{n \in \mathbb{N} : x_n \notin U\}) = \lim_{n\to\infty} \frac{|\{n \in \mathbb{N} : x_n \notin U\}|}{\varrho(n)} = 0.$$

In this paper, we continue our study of the weighted density in order to identify a topological property related to compactness.

2. Preliminaries

This part provides a quick overview of the basic tools and mathematical ideas needed to understand the main conclusions. Unless otherwise indicated, this paper does not presuppose any separation axioms; a space will refer to a topological space in this paper. We refer to [10] for further concepts and symbols.

Definition 2.1. [10] *If each open covering of a space X posses a finite sub-cover, then that space is said to be countably compact.*

Definition 2.2. [10] *A countable family $\mathcal{F} = \{F_s\}_{s\in\mathbb{N}}$ whose elements are subsets of a set X is stated to have finite intersection property (FIP), if $\bigcap_{i=1}^{n} F_{s_i} \neq \emptyset$ and $\mathcal{F} \neq \emptyset$ for every finite set $\{s_1, s_2, s_3, \dots, s_n\} \subseteq \mathbb{N}$.*

Theorem 2.1. [10] *Every collection of closed subsets of a space X having FIP produces non-empty intersection if and only if X is compact.*

Definition 2.3. [8] *A statistical compact (or s-compact) space is a topological space X in which every countable open cover $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$ posses a sub-cover $\mathcal{V} = \{U_{m_k} : k \in \mathbb{N}\}$ for which $\delta\left(\{m_k : U_{m_k} \in \mathcal{V}\}\right) = 0$.*

Definition 2.4. [8] *A countable family $\mathcal{F} = \{F_s\}_{s\in\mathbb{N}}$ whose elements are subsets of a set X is stated to have δ_r-intersection property if $\bigcap_{n\in S} F_n \neq \emptyset$ for every subset $S \subseteq \mathbb{N}$ with $\delta(S) = r$ and $\mathcal{F} \neq \emptyset$.*

Theorem 2.2. [8] *Every collection of closed subsets of a space X having δ_0-intersection property produces non-empty intersection if and only if X is s-compact.*

3. s_ϱ-Compact space

Using the concept of weighted density, we want to find a covering criteria that lies somewhere between countable compactness and statistical compactness.

Definition 3.1. *Let X be a space with the weight function ϱ. If every countable open covering $\mathcal{P} = \{P_n : n \in \mathbb{N}\}$ of X posses a sub cover $\mathcal{Q} = \{P_{n_k} : k \in \mathbb{N}\}$ for which $\delta_\varrho(\{n_k \in \mathbb{N} : P_{n_k} \in \mathcal{Q}\}) = 0$, then X is stated as a weighted statistical compact space (or shortly s_ϱ-compact space).*

Theorem 3.1. *Every countably compact space is an s_ϱ-compact space.*

Proof. Let X be a countably compact space. Then, every countable open cover $\mathcal{P} = \{P_n : n \in \mathbb{N}\}$ has a finite sub cover $\mathcal{Q} = \{P_{n_1}, P_{n_2}, P_{n_3}, \ldots, P_{n_k}\}$. As the sub cover is a finite, the set $\{n_1, n_2, \cdots, n_k\} \subseteq \mathbb{N}$ is finite having weighted density zero. Hence, the space X is an s_ϱ-compact space. □

Example 3.1. *There exists a non-compact, s_ϱ-compact space.*

Let $X = \{(a,b) : a^2 + b^2 < 1\}$ and $\tau = \{A_n = \{(a,b) : a^2 + b^2 < 1 - \frac{1}{n}\} : n \in \mathbb{N}\} \cup \{\emptyset, X\}$. Clearly, X is a topological space. Consider the weight function $\varrho : \mathbb{N} \to [0, \infty)$ such that $\varrho(n) = log(1 + n)$. For an arbitrary countable open cover $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$, we can choose a sub sequence $\mathcal{V} = \{U_{n_k} : k \in \mathbb{N}\}$ such that $U_{n_1} = U_1$ and $U_{n_{k+1}} \supseteq U_{n_k}$ for all $k \in \mathbb{N}$. So $\mathcal{V}$ is an increasing sub sequence of $\mathcal{U}$ that covers X. Now, we choose a sub sequence $\mathcal{W}$ of $\mathcal{V}$ as $\mathcal{W} = \{U_{n_{k^k}} : k \in \mathbb{N}\}$. It is clear that $\bigcup \mathcal{W} = X$ and

$$\delta_\varrho(\{n_k \in \mathbb{N} : U_{n_k} \in \mathcal{W}\}) = \lim_{k\to\infty} \frac{|\{k^k : k \in \mathbb{N}\}|}{log(1 + k^k)}$$

$$= \lim_{k\to\infty} \frac{k}{log(1 + k^k)} = \lim_{k\to\infty} \frac{1 + k^k}{k^k(1 + logk)} = 0.$$

But $\mathcal{W}$ is a sub sequence of $\mathcal{V}$ and $\mathcal{V}$ is a sub sequence of $\mathcal{U}$. So, $\mathcal{W}$ is a sub cover of $\mathcal{U}$ such that $\delta_\varrho(\{n_k \in \mathbb{N} : U_{n_k} \in \mathcal{W}\}) = 0$. Thus, X is an s_ϱ compact space.

Now, consider the open cover $\mathcal{A} = \{A_n = \{(a,b) : a^2 + b^2 < 1 - \frac{1}{n}\} : n \in \mathbb{N}\}$ and if possible suppose that it has a finite sub cover $\mathcal{A}' = \{A_{n_1}, A_{n_2}, A_{n_3}, \ldots, A_{n_k}\}$. We take $n_{max} = max\{n_1, n_2, n_3, \ldots, n_k\}$. Therefore, $\bigcup \mathcal{A}' = A_{n_{max}} = \{(a,b) : a^2 + b^2 < (1 - \frac{1}{n_{max}})\}$. The portion $\{(a,b) : (1 - \frac{1}{n_{max}}) \leq a^2 + b^2 < 1\}$ remains uncovered, which is a contradiction. So, $\mathcal{A}$ can not have a finite sub cover. Thus, (X, τ) is not compact.

Theorem 3.2. *Every s_ϱ-compact space is an s-compact space.*

Proof. Let X be a space having s_ϱ-compactness. Then, every countable open covering $\mathcal{P} = \{P_n : n \in \mathbb{N}\}$ of X posses a sub cover $\mathcal{Q} = \{P_{n_k} : k \in \mathbb{N}\}$ such that $\delta_\varrho(\{n_k \in \mathbb{N} : P_{n_k} \in \mathcal{Q}\}) = 0$. But $\delta(\{n_k \in \mathbb{N} : P_{n_k} \in \mathcal{Q}\}) \leq \delta_\varrho(\{n_k \in \mathbb{N} : P_{n_k} \in \mathcal{Q}\}) = 0$. So, X is a statistical compact space. Hence, every s_ϱ-compact space is an s-compact space. □

Open Problem 3.3. *Does there exists a topological space which is statistical compact but not s_ϱ-compact?*

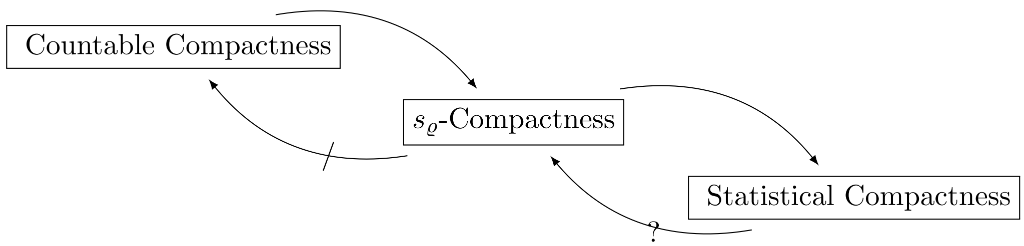

FIGURE 1. The relationship between compactness variations.

Theorem 3.4. *s_ϱ-compactness is a closed hereditary property.*

Proof. Let (X, τ) be a s_ϱ-compact topological space and (B, τ_B) be a closed sub-space of X and let $\mathcal{U} = \{U_n \in \tau_B : n \in \mathbb{N}\}$ be a covering of (B, τ_B).

$$\text{Therefore, } B = \bigcup_{n\in\mathbb{N}} U_n = \bigcup \mathcal{U}.$$

Now, for every $n \in \mathbb{N}$, we can find a τ-open set V_n for which $U_n = B \cap V_n$.

$$\text{Therefore, } B = \bigcup_{n\in\mathbb{N}} U_n \subseteq \bigcup_{n\in\mathbb{N}} V_n.$$

Consider $\mathcal{W} = \{W_n : n \in \mathbb{N}\}$, where

$$W_n = \begin{cases} X \setminus B & \text{if } n = 1, \\ V_{n-1} & \text{otherwise,} \end{cases}$$

So, $\mathcal{W} = \{W_n : n \in \mathbb{N}\}$ is a countably infinite cover of the s_ϱ-compact space X. So, we can find a sub cover $\mathcal{P} = \{W_{n_k} : k \in \mathbb{N}\}$ having $\delta_\varrho(\{n_k : W_{n_k} \in \mathcal{P}\}) = 0$. Let $\mathcal{P}_B = \{B \cap W_{n_k} : k \in \mathbb{N}\}$, then $\mathcal{U}$ have a sub cover $\mathcal{P}_B$ covering B. Now, if $W_1 \notin \mathcal{P}$, then $\{n_k : W_{n_k} \in \mathcal{P}\} = \{n_k : B \cap W_{n_k} \in \mathcal{P}_B\}$ and $\delta_\varrho(\{n_k : W_{n_k} \in \mathcal{P}\}) = \delta_\varrho(\{n_k : B \cap W_{n_k} \in \mathcal{P}_B\}) = 0$. If $W_1 \in \mathcal{P}$, then $|\{n_k : W_{n_k} \in \mathcal{P}\}| = |\{n_k : B \cap W_{n_k} \in \mathcal{P}_B\}| - 1$.

$$\text{So, } \delta_\varrho(\{n_k : W_{n_k} \in \mathcal{P}\}) = \delta_\varrho(\{n_k : B \cap W_{n_k} \in \mathcal{P}_B\}) = 0.$$

Hence, (B, τ_B) is an s_ϱ-compact space.

□

Theorem 3.5. *If $B \subseteq X$ and (B, τ_B) is an s_ϱ-compact closed sub-space of a topological space X, then for every family of open sets $\{W_n : n \in \mathbb{N}\}$ of X such that $B \subseteq \bigcup W_n$ for all $n \in \mathbb{N}$ there exists a subset $P \subseteq \mathbb{N}$ with $\delta_\varrho(P) = 0$ such that $B \subseteq \bigcup_{n\in P} W_n$.*

Proof. Let $\{W_n : n \in \mathbb{N}\}$ be a family of open subsets of X ensuring $\bigcup_{n\in\mathbb{N}} W_n \supseteq B$. Then, $B = \bigcup_{n\in\mathbb{N}}(B \cap W_n)$, which implies that B is covered by $\{B \cap W_n : n \in \mathbb{N}\}$, a collection of τ_B-open sets. Also, (B, τ_B) is an s_ϱ-compact space. Therefore, there exists a set $P \subseteq \mathbb{N}$ with $\delta_\varrho(P) = 0$ such that $B = \bigcup_{n\in P}(B \cap W_n)$. Thus, $B \subseteq \bigcup_{n\in P} W_n$. Hence, for every family of open sets $\{W_n : n \in \mathbb{N}\}$ of X such that $B \subseteq \bigcup W_n$ for all $n \in \mathbb{N}$ there exists a subset $P \subseteq \mathbb{N}$ with $\delta_\varrho(P) = 0$ such that $B \subseteq \bigcup_{n\in P} W_n$. □

Theorem 3.6. *Let W be an open subset of a topological space X and consider the weight function ϱ. If a family $\{G_m : m \in \mathbb{N}\}$ of closed subsets of X consists at least one s_ϱ-compact set (say G_{m_0}) such that $\bigcap_{m\in\mathbb{N}} G_m \subseteq W$, then there exists $P \subseteq \mathbb{N}$ with $\delta_\varrho(P) = 0$ and $\bigcap_{m\in P} G_m \subseteq W \cup G^c_{m_0}$.*

Proof. Let G_{m_0} be an s_ϱ-compact set in the family $\{G_m : m \in \mathbb{N}\}$. As $W \in \tau$ is open, so W^c is closed. Thus, $W^c \cap G_{m_0} = G_{m_0} \setminus W$. Since $G_{m_0} \setminus W \subseteq G_{m_0}$ and G_{m_0} is an s_ϱ-compact set so by Theorem 3.4, $G_{m_0} \setminus W$ is an s_ϱ-compact set. Let $B = G_{m_0} \setminus W$. $\{W_m = X \setminus G_m : m \in \mathbb{N}\}$ is family of open sets.

$$\text{Now, } \bigcup_{m\in\mathbb{N}} W_m = \bigcup_{m\in\mathbb{N}} (X \setminus G_m) = X \setminus \bigcap_{m\in\mathbb{N}} G_m.$$

$$\text{and } X \setminus \bigcap_{m\in\mathbb{N}} G_m \supseteq X \setminus W \supseteq G_{m_0} \setminus W = B.$$

So, $B \subseteq \bigcup_{m\in\mathbb{N}} W_m$. But, $B = G_{m_0} \setminus W$ is an s_ϱ-compact space. Therefore, by Theorem 3.5, there exists $P \subseteq \mathbb{N}$ with $\delta_\varrho(P) = 0$ such that $B \subseteq \bigcup_{m\in P} W_m$. So, $G_{m_0} \setminus W = (X \setminus W) \cap G_{m_0} \subseteq \bigcup_{m\in P} W_m$.

$$\text{Thus, } (X \setminus W) \cap (X \setminus G^c_{m_0}) \subseteq \bigcup_{m\in P} X \setminus G_m = X \setminus \bigcap_{m\in P} G_m.$$

$$\text{Therefore, } X \setminus (W \cup G^c_{m_0}) \subseteq X \setminus \bigcap_{m\in P} G_m.$$

Hence, $\bigcap_{m\in P} G_m \subseteq W \cup G^c_{m_0}$. □

Theorem 3.7. *Let $\{(X_m, \tau_m) : m = 1, 2, \cdots, s\}$ be a finite collection of s_ϱ-compact sub-spaces of X such that $X = \bigcup_{m=1}^{s} X_m$. Then, X is an s_ϱ-compact space.*

Proof. Let (X_m, τ_m) be an s_ϱ-compact sub-space of X for $m = 1, 2, 3, \ldots, s$ such that $X = \bigcup_{m=1}^{s} X_m$ and let $\mathcal{W} = \{W_n : n \in \mathbb{N}\}$ be a countable open cover of X. Then, $\mathcal{W}_m = \{X_m \cap W_n : n \in \mathbb{N}\}$ are countable open covers of (X_m, τ_m) where $m = 1, 2, \cdots, s$. Therefore, there exist $\mathcal{V}_m = \{X_m \cap W_{n_k} : k \in \mathbb{N}\}$ for every $\mathcal{W}_m$ of (X_m, τ_m) such that $\delta_\varrho(\{n_k \in \mathbb{N} : X_m \cap W_{n_k} \in \mathcal{V}_m\}) = 0$.

Now,

$$\delta_\varrho(\bigcup_{m=1}^{s} \{n_k \in \mathbb{N} : X_m \cap W_{n_k} \in \mathcal{V}_m\}) \leq \sum_{m=1}^{s} \delta_\varrho(\{n_k \in \mathbb{N} : X_m \cap W_{n_k} \in \mathcal{V}_m) = 0.$$

Also, $\bigcup_{m=1}^{s} \mathcal{V}_m$ are covers of X. So, $X \subseteq \bigcup_{m=1}^{s} \mathcal{V}_m \subseteq \bigcup_{m=1}^{s} \{W_{n_k} : k \in \mathbb{N}$ and $X_m \cap W_{n_k} \in \mathcal{V}_m\} = \mathcal{P}$. Thus, $\mathcal{P}$ is a sub cover of $\mathcal{W}$ such that $\delta_\varrho(\{n_k : W_{n_k} \in \mathcal{P}\}) = 0$. Hence, X is an s_ϱ-compact space. □

Theorem 3.8. *s_ϱ-compactness is preserved under surjective open continuous mapping.*

Proof. Consider a surjective map $f : (X, \tau) \longrightarrow (Y, \sigma)$ which is both open and continuous, where X is an s_ϱ-compact space, ϱ being a weight function.

Suppose that $\mathcal{A} = \{A_n\}_{n \in \mathbb{N}}$ is a random covering of Y, $\mathcal{A}$ being countable and elements of $\mathcal{A}$ being open. Then, $Y = \bigcup_{n \in \mathbb{N}} A_n$. So, $f^{-1}(Y) = f^{-1}(\bigcup_{n \in \mathbb{N}} A_n)$. Thus, $X = \bigcup_{n \in \mathbb{N}} f^{-1}(A_n)$. So, X is covered by $\{f^{-1}(A_n) : n \in \mathbb{N}\}$, which is an open covering (f being continuous and $\mathcal{A} = \{A_n\}_{n \in \mathbb{N}}$ being open). Also, X is an s_ϱ-compact space. Therefore, we will get a countable sub covering $\{f^{-1}(A_{n_k})\}_{k \in \mathbb{N}}$ of X having $\delta_\varrho(\{n_k : k \in \mathbb{N}\}) = 0$. Therefore, $\bigcup_{n_k \in \mathbb{N}} \{f^{-1}(A_{n_k})\} = X$ that implies $f(\bigcup_{n_k \in \mathbb{N}} \{f^{-1}(A_{n_k})\}) = f(X) = Y$. i.e., $Y = \bigcup_{n_k \in \mathbb{N}} \{A_{n_k}\}$. Thus, Y is covered by $\{A_{n_k}\}_{k \in \mathbb{N}}$ which is a subset of $\{A_n\}_{n \in \mathbb{N}}$ with $\delta_\varrho(\{n_k : k \in \mathbb{N}\}) = 0$.

Hence, Y also have the s_ϱ-compactness property. □

Under the effect of weighted statistical density, now we search for a finite intersection like attributes for s_ϱ compactness.

Definition 3.2. *A countable family $\mathcal{D} = \{D_n : n \in \mathbb{N}\} \subseteq \mathcal{P}(X)$ is stated to posses ${}_\varrho\Delta_r$-intersection property (${}_\varrho\Delta_r$-IP) if $\mathcal{D} \neq \emptyset$ and for each $P \subseteq \mathbb{N}$ having $\delta_\varrho(P) = r$, gives $\bigcap_{n \in P} D_n \neq \emptyset$.*

Theorem 3.9. *Every countable collection of closed subsets of X having ${}_\varrho\Delta_0$-IP produces non-empty intersection if the space X is s_ϱ-compact and vice versa.*

Proof. Let X be a space having s_ϱ-compactness and $\mathcal{D} = \{D_n : n \in \mathbb{N}\}$ be an arbitrary family of closed subsets of X with ${}_\varrho\Delta_0$-intersection property .

If possible, let us consider $\bigcap_{n \in \mathbb{N}} D_n = \emptyset$ and $F_n = X \setminus D_n$. Then, $X = X \setminus \bigcap_{n \in \mathbb{N}} D_n =$

$\bigcup_{n\in\mathbb{N}} X\setminus D_n = \bigcup_{n\in\mathbb{N}} F_n$. Therefore, X is covered by $\{F_n\}_{n\in\mathbb{N}}$, elements of the collection $\{F_n\}_{n\in\mathbb{N}}$ being open sets. But X is an s_ϱ-compact space. Thus, we can obtain a $P\subseteq\mathbb{N}$ having $\delta_\varrho(P)=0$ and $\bigcup_{n\in P} F_n = X$. Now,

$$X=\bigcup_{n\in P} F_n=\bigcup_{n\in P} X\setminus D_n = X\setminus \bigcap_{n\in P} D_n.$$

Therefore, $\bigcap_{n\in P} D_n=\emptyset$, that leads to a contradiction. So, $\mathcal{D}=\{D_n : n\in\mathbb{N}\}$ has ${}_g\Delta_0$-intersection property.

Conversely, let $\mathcal{W}=\{W_n : n\in\mathbb{N}\}$ be a countable open cover of X. Now, $\mathcal{D}=\{D_n = X\setminus W_n : n\in\mathbb{N}\}$ is a countable family of closed sets. Now,

$$\bigcap_{n\in\mathbb{N}} D_n=\bigcap_{n\in\mathbb{N}}(X\setminus W_n)=X\setminus\bigcup_{n\in\mathbb{N}} W_n=\emptyset.$$

By contra positive process of our assumption it does not have ${}_\varrho\Delta_0$-intersection property. Therefore, there exists $P\subseteq\mathbb{N}$ with $\delta_\varrho(P)=0$ and $\bigcap_{n\in P} D_n=\emptyset$. So, $\bigcap_{n\in P}(X\setminus W_n)=\emptyset$ i.e., $X\setminus\bigcup_{n\in P} W_n=\emptyset$. Thus, $X=\bigcup_{n\in P} W_n$.

Hence, X is an s_ϱ-compact space. □

4. Conclusion remarks

s_ϱ compact space serves as an intermediate between countable compactness and statistical compactness. This compactness property is preserved under closed sub-space and open continuous surjection. s_ϱ compactness can be characterized in terms of families of closed sets by means of ${}_\varrho\Delta_0$ intersection property.

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