International Journal of Maps in Mathematics

*Volume **8**, Issue **1**, 2025, Pages:**247-257***

E-ISSN: 2636-7467

www.simadp.com/journalmim

A CLASS OF INDEFINITE ALMOST PARACONTACT METRIC MANIFOLDS

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Abstract. This research, we develop a new class of indefinite almost paracontact metric manifolds, termed (ϵ)-para Kenmotsu manifolds and we obtain some typical identities for the curvature tensor, scalar curvature and Ricci tensor. Furthermore, in particular, we investigate the curvature features of *three*-dimensional (ϵ)-para Kenmotsu manifolds. We establish an essential as well as sufficient condition for an (ϵ)-para Kenmotsu 3-manifold to have an indefinite space form. Furthermore, we classify and demonstrate that (ϵ)-para Kenmotsu 3-manifolds, which are either semi-symmetric, Ricci-semi-symmetric or semi-symmetric type, are η-Einstein. In conclusion, we create a 3-D (ϵ)-para Kenmotsu manifold example.

Keywords: Indefinite almost paracontact metric manifold, Ricci semi-symmetric manifold, (ϵ)-para Kenmotsu manifold, semi-symmetric and η-Einstein manifolds.

2010 Mathematics Subject Classification: 53C25, 53C50.

1. Introduction

With an emphasis on Sasakian manifolds, Takahashi [16] introduced almost contact manifolds equipped with pseudo-Riemannian metrics in 1969. The terms (ϵ)-almost contact metric and (ϵ)-Sasakian have also been used to refer to indefinite almost contact metric manifolds and the indefinite Sasakian manifolds, respectively. The (ϵ)-Kenmotsu manifold which has

Received:2024.07.24 *Revised:2024.09.22* *Accepted:2024.12.04*

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been introduced by De and Sarkar [1] is based on a class of almost contact Riemannian manifolds called Kenmotsu manifolds [3]. They proved that the curvatures are influenced by the presence of a new structure with indefinite metrics.

On the other hand, in 1976, Sato [12] defined the notions of an almost paracontact structure, which is similar to the almost contact structure. By replacing the vector field ξ in almost paracontact manifold with $-\xi$, Matsumoto [4] first proposed the concept of Lorentzian almost paracontact in 1989. Lorentzian para-Sasakian (LP-Sasakian) manifolds connected to the Lorentzian metric are the outcome of this. While the structural vector field ξ is always time-like, the semi-Riemannian metric in a Lorentzian almost paracontact manifold has only an index of 1. Abdul Haseeb along with Rajendra Prasad [2] defined Lorentzian para-Kenmotsu (also called LP-Kenmotsu) manifolds in 2018. Afterward, numerous geometers, including [7, 8, 9, 10, 11, 14, 18], have extensively investigated these manifolds.

Inspired by these studies, Tripathi *et al.*, [17] presented the notion of an indefinite almost paracontact metric structure, also referred to as an (ϵ)-almost paracontact structure, by linking an almost paracontact structure with a semi-Riemannian metric, which need not be Lorentzian. In this instance, $\epsilon = 1$ or $\epsilon = -1$ indicates that the structure vector field ξ is either space-like or time-like. In addition, they introduced and examined the characteristics of (ϵ)-para Sasakian [17] and (ϵ)-para Sasakian 3-manifolds [6].

Inspired by the prior study, the current paper continues the discussion of indefinite almost paracontact metric manifolds, introducing the idea of (ϵ)-para Kenmotsu manifolds based on para-Kenmotsu manifolds, defined by Sinha and Sai Prasad in 1995 [13].

The format of the paper is as follows: We define an (ϵ)-para Kenmotsu manifold, investigate some of its fundamental characteristics and derive some typical identities for the Ricci tensor, scalar curvature, and curvature tensor in Section-2. Furthermore, we explore the curvature features of (ϵ)-para Kenmotsu three-dimensional manifolds. We attained an essential as well as sufficient condition for an (ϵ)-para Kenmotsu 3-dimensional manifold M_3 to have an indefinite space form. Furthermore, in Sections 3, 4, and 5, we classify and demonstrate that (ϵ)-para Kenmotsu 3-manifolds, which are either semi-symmetric, Ricci-semi-symmetric, or semi-symmetric type, are η-Einstein. In conclusion, we create a 3-D (ϵ)-para Kenmotsu manifold example.

2. (ϵ)-para Kenmotsu manifolds

A differentiable manifold (M_n, g) of n-dimension is regarded as an (ϵ)-almost paracontact metric manifold [17] with the structure tensors $(\phi,\ \xi,\ \eta,\ g,\ \epsilon)$, where the tensor field $(1,1)$ is represented by ϕ, the vector field by ξ, the 1-form by η, the semi-Riemannian metric by $g(X,Y)$, not necessarily Lorentzian, such that

$$\eta(\xi) = 1, \tag{2.1}$$

$$\overline{\overline{X}} \ = \ X - \eta(X)\xi, \ where \ \overline{X} = \phi X, \tag{2.2}$$

$$g\ (\xi,\ \xi) = \epsilon, \tag{2.3}$$

$$g\ (X,\ \xi) = \epsilon\ \eta(X), \tag{2.4}$$

$$g\ (\phi X,\ \phi Y) = g\ (X,\ Y) - \epsilon\ \eta(X)\ \eta(Y); \tag{2.5}$$

for every $X, Y \in \chi(M_n)$, and $\chi(M_n)$ is a collection of differentiable vector fields on M_n. Since the structure vector field ξ which has been vector field that is either space-like or time-like, and the rank of that tensor filed ϕ is $(n-1)$, in this case, (ϵ) is either 1 or -1.
If $g(X,\ Y)$ is positive definite, that is

$$d\eta(X,\ Y) = g(X,\ \phi Y), \tag{2.6}$$

then the manifold M_n is referred as an almost paracontact metric manifold [12]. Evidently, on M_n, we have

$$\phi\ \xi = 0,\ \ \eta(\phi X) = 0. \tag{2.7}$$

Definition 2.1. *An (ϵ)-paracontact metric structure is referred to as an (ϵ)-para Kenmotsu structure if*

$$(\nabla_X \Phi)Y = g\ (X, \phi Y)\xi - \epsilon\ \eta(Y)\phi X, \tag{2.8}$$

where, for all vector fields X and Y, the Levi-Civita connection is given by ∇ with regard to the indefinite metric $g(X,\ Y)$. An (ϵ)-para Kenmotsu manifold is a manifold M_n with the (ϵ)-para Kenmotsu structure.

For $\epsilon = 1$ and the Riemannian metric $g(X,\ Y)$, the manifold M_n is the standard para-Kenmotsu manifold.

An (ϵ)-almost paracontact metric manifold is an (ϵ)-para Kenmotsu manifold if and only if

$$\nabla_X \xi = \epsilon\ \phi^2(X) = \epsilon\ (X - \eta(X)\xi). \tag{2.9}$$

Furthermore, from (2.4), we get

$$(\nabla_X \eta)\ Y = \epsilon\ g(\nabla_X \xi, Y).$$

Then by using the above expression and (2.9), we have

$$(\nabla_X \eta)\ Y = \epsilon\ g(X,\ Y) - \eta(X)\ \eta(Y). \tag{2.10}$$

Lemma 2.1. *Let M_n be an (ϵ)-para Kenmotsu manifold. Then, the type $(1,3)$ Riemannian Christoffel curvature tensor $R(X,Y)$ satisfies*

$$R\ (X,\ Y)\xi = \ \eta(X)\ Y - \eta(Y)\ X. \tag{2.11}$$

Consequently,

$$R\ (\xi,\ X)\ Y = \epsilon\ \eta(Y)X\ -\ g(X,\ Y)\ \xi, \tag{2.12}$$

$$R\ (\xi,\ X)\xi = \ \epsilon X - \epsilon\ \eta(X)\ \xi, \tag{2.13}$$

$$\eta(R(X,Y)\ Z) = \ \epsilon\ g(X,\ Z)\ \eta(Y) - \epsilon\ \eta(X)\ g(Y,\ Z), \tag{2.14}$$

$$S\ (Y,\ \xi) = \ -(n-1)\ \eta(Y), \tag{2.15}$$

for all vector fields X, Y and Z, where $S(X,Y)$ denotes the Ricci tensor and Q is known to be the Ricci operator with regard to ∇.

Proof. By using the equations (2.9), (2.1), and (2.10) in

$$R(X,\ Y)\ \xi = \ \nabla_X \nabla_Y \xi - \nabla_Y \nabla_X \xi - \nabla_{[X,\ Y]} \xi,$$

we obtain (2.11). Moreover, we have

$$R(X,Y,Z,W) = \ g(X,\ Z)\ g(Y,\ W) - g(Y,\ Z)\ g(X,\ W).$$

Then, by using (2.4) and from the above expression, we obtain the results (2.12), (2.13), and (2.14). Further, on the contraction of the above expression with respect to X and W, we get (2.15), and hence it completes the proof. □

Furthermore, it is recognized that we have in a semi-Riemannian 3-manifold

$$\begin{aligned} R\ (X,\ Y)Z = g(X,\ Z)\ QY - g(Y,\ Z)\ QX\ +\ S(X,\ Z)Y \\ - S(Y,\ Z)X - \frac{r}{2}\ [g(X,\ Z)Y\ -\ g(Y,\ Z)X], \end{aligned} \tag{2.16}$$

where r is the manifold's scalar curvature.

By substituting ξ for Z in (2.16) as well as utilizing the equation (2.11) for $n = \ 3$, we have

$$\epsilon\ [\eta(Y)\ QX - \eta(X)\ QY] = \left[3 + \frac{r\epsilon}{2}\right]\ [\eta(Y)\ X - \eta(X)\ Y]. \tag{2.17}$$

Then for $Y = \xi$ in (2.17) and utilizing (2.2) & (2.15), we get

$$QX = \frac{1}{2}(r + 6\epsilon)\ X - \frac{1}{2}(r + 10\epsilon)\ \eta(X)\ \xi,$$

and hence

$$S(X,\ Y) = g(QX,\ Y)\ =\ \frac{1}{2}\big[(r + 6\epsilon)\ g(X,\ Y) - \ \epsilon\ (r + 10\epsilon)\ \eta(X)\ \eta(Y)\big]. \tag{2.18}$$

Therefore from (2.16) and (2.18)

$$\begin{aligned} R(X, Y)\ Z = &\big[g(X,\ Z)\ Y\ -\ g(Y,\ Z)\ X\big]\Big[\frac{r}{2}\ +\ 6\epsilon\Big] \\ &+\ \big[g(Y,\ Z)\ \eta(X)\ \xi\ -\ g(X,\ Z)\ \eta(Y)\ \xi\ +\ \epsilon\eta(Y)\ \eta(Z)X - \epsilon\eta(X)\ \eta(Z)Y\big]\Big[\frac{r}{2} + 5\epsilon\Big]. \end{aligned} \tag{2.19}$$

It demonstrates that an (ϵ)-para Kenmotsu manifold with constant scalar curvature is an indefinite space form.

Lemma 2.2. *If the scalar curvature of an (ϵ)-para Kenmotsu manifold of dimension* 3 *is* -6ϵ*, then the manifold has an indefinite space form. Also, the converse.*

Proof. Consider a 3-D (ϵ)-para Kenmotsu manifold M_3 which has an indefinite space form. Then

$$R(X,\ Y)\ Z = c\ [g(X,\ Z)\ Y - g(Y,\ Z)\ X], \tag{2.20}$$

where c represents the manifold's constant curvature. Using the definition of Ricci curvature as well as equation (2.20), we get

$$S(X,\ Y)\ =\ 2c\ g(X,\ Y). \tag{2.21}$$

Utilizing (2.21) in the scalar curvature definition yields

$$r =\ 6c. \tag{2.22}$$

Next, it is evident from (2.21) and (2.22) that

$$S(X,\ Y) = \frac{r}{3}\ g(X,\ Y). \tag{2.23}$$

Using (2.23) and entering $X =\ Y =\ \xi$ in (2.18), we get

$$r =\ -6\epsilon. \tag{2.24}$$

On the other hand, the proof is completed if $r = -6\epsilon$, in which case the manifold is clearly an indefinite space form as shown by equation (2.19). □

Theorem 2.1. *Each (ϵ)-para Kenmotsu manifold of dimension* 3 *is* η*-Einstein.*

Proof. The theorem's proof is derived from (2.18) and (2.11). □

3. Semi-symmetric (ϵ)-para Kenmotsu 3-manifolds

Definition 3.1. *An (ϵ)-para Kenmotsu manifold of dimension* 3 *is semi-symmetric* [15] *if*

$$R(X,\ Y)\cdot R\ =\ 0, \tag{3.25}$$

holds for all vector fields X and Y.

Theorem 3.1. *M_3 is an η-Einstein manifold, if it is a semi-symmetric (ϵ)-para Kenmotsu* 3*-manifold.*

Proof. Consider

$$(R(X,Y)\cdot R)\ (Z,W,U) = \ 0, \tag{3.26}$$

for all vector fields $X,\ Y,\ Z$, and U.

The above equation implies that

$$\begin{aligned}(R(X,\ Y, R\ (Z,\ W,\ U))\ -\ R(R(X,\ Y,\ Z),\ W,\ U)\\ - R(Z,\ R(X,\ Y,\ W), U)\ -\ R(Z,\ W, R(X,\ Y)U) = 0.\end{aligned} \tag{3.27}$$

Afterward, specifically for $X = \xi$, we have

$$\begin{aligned}(R(\xi,\ Y, R\ (Z,\ W,\ U))\ -\ R(R(\xi,\ Y,\ Z),\ W,\ U)\\ -\ R(Z,\ R(\xi,\ Y,\ W), U)\ -\ R(Z,\ W, R(\xi,\ Y)U) = 0.\end{aligned} \tag{3.28}$$

Using the aforementioned equation along with (2.12) and (2.14), we now obtain

$$\begin{aligned}{}'R\ (Z,\ W, U,\ Y)\xi = \ &\epsilon\, g(Z,\ U)\ \eta(W)Y -\ \epsilon\, g(W,\ U)\ \eta(Z)Y -\ \epsilon\, \eta(Z)\ R(Y,\ W,\ U)\\ &+\ g(Y,Z)\ R(\xi,\ W,\ U) -\ \epsilon\, \eta(W)\ R(Z,\ Y,\ U) +\ g(Y,\ W)\ R(Z,\ \xi,\ U)\\ &- \epsilon\, \eta(U)\ R(Z,\ W,\ Y)\ +\ g(Y,\ U)\ R(Z,\ W,\ \xi).\end{aligned} \tag{3.29}$$

Then by using equations (2.11), (2.12), (2.14), and the inner product with ξ, the above equation is reduced to

$${}'R(Z,\ W,\ U,\ Y)\ =\ g(Y,\ W)\ g(Z,\ U)\ - g(Y,\ Z)\ g(W,\ U), \tag{3.30}$$

which, when contracted with regard to U and W, results in

$$S(Y,\ Z) = \ \eta(Y)\ \eta(Z) - n\ \epsilon\ g(Y,Z). \tag{3.31}$$

For $Z = \xi$ in (3.31), we obtain

$$S(Y,\ \xi) = -(n-1)\ \eta(Y). \tag{3.32}$$

This concludes the proof of the theorem. □

4. Ricci semi-symmetric (ϵ)-para Kenmotsu 3-manifolds

If a semi-Riemannian manifold, M_n, satisfies the following condition, its Ricci tensor, S, is deemed Ricci-semi-symmetric [5].

$$R\ (X,\ Y)\cdot S = 0,\ for\ all\ X, Y \in \chi(M_n), \tag{4.33}$$

where $R(X,\ Y)$ serves as a derivation on S.

Let us suppose that M_3 be a Ricci-semi-symmetric (ϵ)-para Kenmotsu *three*-dimensional manifold. That is

$$(R\ (X,Y)\cdot S)\ (Z,\ U) = 0. \tag{4.34}$$

The above equation further implies that

$$S(R(X,\ Y)Z,\ U)\ +\ S(U,\ R(X,\ Y)\ Z) = 0. \tag{4.35}$$

For $X = \xi$ in (4.35), we have

$$S\ (R(\xi,\ Y)Z,\ U)\ +\ S(U,\ R(\xi,\ Y)\ Z) = 0. \tag{4.36}$$

Now by using (2.12) and (2.15), we have, from the above equation

$$\epsilon\ \eta(Z)\ S(Y,\ U)\ +\ (n-1)\ g(Y,\ Z)\ \eta(U) + \epsilon\ \eta(U)\ S(Y,\ Z) + (n-1)\ g(Y,\ U)\ \eta(Z) = 0. \tag{4.37}$$

Using equations (2.2) and (2.4) and substituting $U = Z = \xi$ in (4.37), we obtain

$$S(Y,\ \xi) = -(n-\ 1)\ \eta(Y). \tag{4.38}$$

Based on this, we could say the following:

Theorem 4.1. *M_3 is an η-Einstein manifold, if it is a Ricci-semi-symmetric (ϵ)-para Kenmotsu 3-manifold.*

5. Semi-symmetric type (ϵ)-para Kenmotsu 3-manifolds

A semi-Riemannian manifold M_n is considered semi-symmetric type if

$$S(X,\ Y)\cdot R=\ 0, \tag{5.39}$$

holds for all vector fields X and Y.

Theorem 5.1. *The semi-symmetric type (ϵ)-para Kenmotsu 3-manifold is η-Einstein.*

Proof. Let M_3 be a semi-symmetric type (ϵ)-para Kenmotsu 3-manifold. Then

$$(S(X,\ Y)\cdot R)(Z,\ U,\ V)=\ 0, \tag{5.40}$$

for all vector fields $X,\ Y,\ Z,\ U$, and V.

The above equation implies that

$$\begin{aligned} &S\ (Y,\ R(Z,\ U,\ V))X-\ S(X,\ R(Z,\ U,\ V))Y+\ S(Y,\ Z)\ R(X,\ U,\ V)\\ &-\ S(Z,\ X)\ R(Y,\ U,\ V)+\ S(Y,\ U)\ R(Z,\ X,\ V)-\ S(U,\ X)\ R(Z,\ Y,\ V)\\ &+\ S(V,\ Y)\ R(Z,\ U,\ X)-\ S(V,\ X)\ R(Z,\ U,\ Y)=\ 0. \end{aligned} \tag{5.41}$$

For $X=\ \xi$ in (5.41), we have

$$\begin{aligned} &S(Y,\ R(Z,\ U,\ V))\xi-\ S(\xi,\ R(Z,\ U,\ V))Y+\ S(Y,\ Z)\ R(\xi,\ U,\ V)\\ &-\ S(Z,\ \xi)\ R(Y,\ U,\ V)+\ S(Y,\ U)\ R(Z,\ \xi,\ V)-\ S(U,\ \xi)\ R(Z,\ Y,\ V)\\ &+\ S(V,\ Y)\ R(Z,\ U,\ \xi)-\ S(V,\ \xi)\ R(Z,\ U,\ Y)=0. \end{aligned} \tag{5.42}$$

Taking the inner product with ξ and using equations (2.22), (2.12), (2.14), (2.15) in (5.42), we get

$$\begin{aligned} &S(Y,R(Z,U,V))+2(n-1)g(Z,V)\eta(U)\eta(Y)-2(n-1)g(U,V)\eta(Y)\eta(Z)\\ &+\epsilon\eta(V)\eta(U)S(Y,Z)-g(U,V)S(Y,Z)+g(Z,V)S(Y,U)\\ &-\epsilon S(Y,U)\eta(V)\eta(Z)+(n-1)g(Z,Y)\eta(U)\eta(V)-(n-1)g(U,Y)\eta(V)\eta(Z)=0. \end{aligned} \tag{5.43}$$

If we put ξ in place of V in (5.43) and on using (2.11), we get

$$\epsilon\ \eta(Z)\ S(Y,\ U)-\epsilon\ \eta(U)\ S(Y,\ Z)\ +\ (n-1)\ g(Y,\ Z)\eta(U)\ -\ (n-1)\ g(U,\ Y)\ \eta(Z)=\ 0. \tag{5.44}$$

Put $U\ =\ Y\ =\ \xi$ in (5.44). Then by using (2.2), (2.4), we get

$$S\ (Z,\ \xi)\ =\ -(n-1)\ \eta(Z), \tag{5.45}$$

which proves the theorem. □

6. Example of a 3-dimensional (ϵ)-para Kenmotsu manifold

In this section, we create a 3-D (ϵ)-para Kenmotsu manifold example.

Example 6.1. *Let $M_3 = \{(x, y, z) \in R^3\}$, where (x, y, z)-represent the standard coordinates in R^3, be a 3-D manifold. Let e_1, e_2, and e_3 be the vector fields on M_3, given by*

$$e_1 = -x\frac{\partial}{\partial x},\ e_2 = x\frac{\partial}{\partial y},\ e_3 = x\frac{\partial}{\partial z}. \tag{6.46}$$

Clearly, at any point in M_3, $\{e_1,\ e_2,\ e_3\}$ represent a set of linearly independent vectors. The Riemannian metric $g(X,\ Y)$ is explained by

$$g\ (e_i, e_j) = \begin{cases} \epsilon, & if\ i = j \\ 0, & if\ i \neq j;\ i,\ j = 1, 2, 3. \end{cases}$$

Let η be the 1-form defined by:

$$g(X, e_1) = \epsilon\ \eta(X).$$

Let ϕ be a $(1,\ 1)$-tensor field on M_3 explained by:

$$\phi(e_1) = 0,\ \phi(e_2) = -\epsilon\ e_2,\ \phi(e_3) = -\epsilon\ e_3.$$

Then the linearity of ϕ & $g(X,\ Y)$ yields that

$$\eta(e_1) = 1,\ \phi^2(X) = X - \eta(X)e_1\ ;$$

$$and\ g(\phi X,\ \phi Y) = g(X,\ Y) - \epsilon\ \eta(X)\ \eta(Y),$$

for all $X,\ Y,\ Z \in M_3$.

The structure $(\phi,\ \xi,\ \eta,\ g,\ \epsilon)$ therefore establishes an (ϵ)-almost paracontact structure on M_3 for $e_1 = \xi$.

Now from (6.46), we also have

$$[e_1,\ e_2] = -\epsilon\ e_2,\ [e_1,\ e_3] = -\epsilon\ e_3,\ [e_2,\ e_3] = 0.$$

Koszul's formula provides the Levi-Civita connection ∇ of the metric tensor $g(X,\ Y)$ as follows:

$$\begin{aligned} 2g\ (\nabla_X Y, Z) &= Xg\ (Y, Z) + Yg\ (Z, X) - Zg\ (X, Y) \\ &\quad - g\ (X, [Y, Z]) - g\ (Y, [X, Z]) + g\ (Z, [X, Y]). \end{aligned} \tag{6.47}$$

Utilizing the above formula and $e_1 = \xi$ yields the following result:

$$\begin{aligned}&\nabla_{e_1}\ e_1 = 0,\ \nabla_{e_1}\ e_2 = 0,\ \nabla_{e_1}\ e_3 = 0;\\ &\nabla_{e_2}\ e_1 = \epsilon\ e_2,\ \nabla_{e_2}\ e_2 = -\epsilon\ e_1,\ \nabla_{e_2}\ e_3 = 0;\\ &\nabla_{e_3}\ e_1 = \epsilon\ e_2,\ \nabla_{e_3}\ e_2 = 0,\ \nabla_{e_3}\ e_3 = -\epsilon\ e_1.\end{aligned} \tag{6.48}$$

The preceding computations show that the manifold M_3 under consideration meets the conditions $\nabla_X\ \xi = \epsilon\ (X -\ \eta(X)\ \xi)$, for all $e_1 =\ \xi$.

It can be seen from this that the manifold M_3, that is being studied is a dimension three (ϵ)-para Kenmotsu manifold having the structure $(\phi,\ \xi,\ \eta,\ g,\ \epsilon)$.

7. Conclusion

This paper defines a new class of indefinite almost paracontact metric manifolds, termed (ϵ)-para Kenmotsu manifolds, using a semi-Riemannian metric. When these manifolds are semi-symmetric or Ricci-semi-symmetric, the metric described by them is both geometrical and physical in nature. The geometrical features of these manifolds are widely applied in a variety of physical and geometrical fields, including the construction of super resolution sensors in electronic and communication systems, in electrical engineering, and in the general theory of relativity.

Acknowledgements. The G. V. P. College of Engineering for Women in Visakhapatnam's Senior Assistant professor, Dr. A. Kameswara Rao, is appreciated by the authors for his insightful advice during the manuscript's preparation. The authors also heartily thank all the reviewers for their valuable suggestions on the paper.

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