International Journal of Maps in Mathematics

Volume ***5****, Issue* ***2****, 2022, Pages:****101-111***

ISSN: 2636-7467 (Online)

www.journalmim.com

CERTAIN RESULTS OF RICCI SOLITONS ON (LCS) MANIFOLDS‡

MUNDALAMANE MANJAPPA PRAVEENA, MALLANNARA SIDDALINGAPPA SIDDESHA *, AND CHANNABASAPPA SHANTHAPPA BAGEWADI

Abstract. In the present paper, we study Ricci solitons of (LCS)-manifolds when quasi-conformal and pseudo projective curvature tensors of (LCS)-manifolds are irrotational and flat. It is revealed that the results obtained by the above methods and using semi-symmetry and Eisenhart problems are the same.

Keywords: (LCS)-manifolds, Ricci soliton, Einstein manifold.

2010 Mathematics Subject Classification: 53B30, 53C15.

1. Introduction

Riemannian geometry gives the study of Riemannian manifolds and Riemannian manifold is equipped with symmetric bilinear and positive definite metric. The Lorentzian manifold is a special case of pseudo-Riemannian manifold which is generalized Riemannian manifold and need not have positive metric tensor.

‡This paper is dedicated to late Prof. K. S. Amur.

Received: 2021.09.24 *Accepted:2021.12.05*

* Corresponding author

Mundalamane M. Praveena; mmpraveenamaths@gmail.com; https://orcid.org/0000-0002-5981-0687

Mallannara S. Siddesha; mssiddesha@gmail.com; https://orcid.org/0000-0003-2367-0544

Channabasappa S. Bagewadi; prof_bagewadi@yahoo.co.in; https://orcid.org/0000-0002-4628-1608

.

The study of Lorentzian concircular structure manifold shortly (LCS)-manifold initiated through Shaikh [15] and Baishya [16] in 2003 and 2005 generalize the idea of LP-Sasakian manifolds inaugurated through Matsumoto (1989) [8], Mihai and Rosca (1992) [9].

Ricci flow was initiated by Hamilton in 1982 and he observed that it's an attractive mathematical model for analyzing the fabrication of the manifold. This is a rule that defaces the metric of a Riemannian manifold in an approach similar to the dissemination of heat. This process is known as the geometrization conjecture of Thurston smoothing out irregularities in the metric. It gives an understanding of the geometry and topology of the manifold. Ricci soliton is self-similar to the Ricci flow(it is extension of Einstein metric) and it is denoted on (M, g) by

$$(L_\vartheta g)(U,W) + 2S(U,W) + 2\Upsilon g(U,W) = 0 \tag{1}$$

ϑ is a vector field created by $\{\phi_t\}_{t\in R}$ one parameter group of transformations, L_ϑ means the Lie derivative along ϑ, Υ is scalar. The Ricci soliton is shrinking: $\Upsilon < 0$, steady: $\Upsilon = 0$ and expanding: $\Upsilon > 0$.

During the current two decades, many mathematicians have investigated Ricci solitons of contact and Kähler manifolds [[13], [14]]. In particular, Praveena et. al. investigated [11, 12] a study on Ricci solitons in generalized complex space form. Hui et. al. [6, 4] and Blaga [3] have studied some classes of Ricci solitons in (LCS)-manifolds. The scholars Bagewadi et. al. [2] studied geometry of Ricci solitons in (LCS)-manifolds. Prompted by the earlier investigations in this article we investigate Ricci Solitons of (LCS)-manifolds when quasi-conformal and pseudo-projective curvature tensors in these manifolds are irrotational and flat. We also study compare our results with Ricci solitons of Eisenhart problem and semi symmetric.

2. PRELIMINARIES

A Lorentzian manifold M together with unit timelike concircular vector field ξ $(g(\xi,\xi) = -1)$, its associated 1-form η $(g(X,\xi) = \eta(X))$ and a $(1,1)$ tensor field $\phi(take\ \ \phi U = \frac{1}{\alpha}\nabla_U \xi)$ is said to be a Lorentzian concircular structure manifold (briefly, (LCS)-manifold). Especially, if we take $\alpha = 1$ then we can obtain the LP-Sasakian structure of Matsumoto in (LCS)-manifold for $(n > 2)$. Moreover in (LCS)-manifold the following relations hold: [15, 16].

$$\phi = I + \eta \otimes \xi, \;\; \eta(\xi) = -1,$$

$$\phi\xi = 0, \;\; \eta \cdot \phi = 0, \;\; g(X, \phi Y) = g(\phi X, Y),$$

$$(\nabla_X \eta)(Y) = \alpha[g(X, Y) + \eta(X)\eta(Y)], \;\; \alpha \neq 0,$$

$$\nabla_X \xi = \alpha[X + \eta(X)\xi] \tag{2}$$

$$\nabla_X \alpha = X\alpha = d\alpha(X) = \rho\eta(X), \;\; \rho = -\xi\alpha = -\xi \cdot \nabla\alpha \tag{3}$$

$$g(\phi U, \phi V) = g(U, V) + \eta(U)\eta(V), \;\; g(U, \xi) = \eta(U),$$

$$R(U, V)\xi = (\alpha^2 - \rho)[\eta(V)U - \eta(U)V], \tag{4}$$

$$R(\xi, U)\xi = (\alpha^2 - \rho)[\eta(U)\xi + U], \tag{5}$$

$$R(\xi, U)V = (\alpha^2 - \rho)[g(U, V)\xi - \eta(V)U],$$

for $U, V \in T(M)$.

3. RICCI SOLITONS OF IRROTATIONAL QUASI-CONFORMAL CURVATURE TENSORS

Yano and Sawaki in 1968 [17] defined and studied a quasi-conformal curvature tensor field $\bar{Q}$ on M of dimension n which includes conformal, concircular and M-projective curvature tensors as specific cases. It is given by

$$\begin{aligned} \bar{Q}(V, U)W &= aR(V, U)W + b[S(U, W)V - S(V, W)U + g(U, W)QV - g(V, W)QU] \\ &\quad -\frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(U, W)V - g(V, W)U], \end{aligned} \tag{6}$$

where $S(V, W) = g(QV, W)$.

Using (2) in $(L_\xi g)(U, W)$ we produce

$$(L_\xi g)(U, W) = 2\alpha[g(U, W) - \eta(U)\eta(W)]. \tag{7}$$

((ξ, Υ, g) is a Ricci soliton in (LCS) manifold.)

Again using (7) and (2) we have

$$S(U, W) = -[(\alpha + \Upsilon)g(U, W) + \alpha\eta(U)\eta(W)]. \tag{8}$$

The preceding equating yields that

$$QU = -[(\alpha + \Upsilon)U + \alpha\eta(U)\xi], \tag{9}$$

$$\text{i.e.,} \qquad S(U, \xi) = -\Upsilon\eta(U), \tag{10}$$

$$r = -\Upsilon n - \alpha(n-1). \tag{11}$$

Put $W = \xi$ in (6) and using (4), (8) we have

$$\bar{Q}(V, U)\xi = A[\eta(U)V - \eta(V)U], \tag{12}$$

where $A = a(\alpha^2 - \rho) - b(2\Upsilon + \alpha) - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)$. The rotation (curl) of quasi-conformal curvature tensor $\bar{Q}$ on a Riemannian manifold is given by

$$\begin{aligned} Rot\ \bar{Q} = Curl\ \bar{Q} = \quad & (\nabla_X\bar{Q})(V, U, W) + (\nabla_V\bar{Q})(X, U, W) \\ & +(\nabla_U\bar{Q})(X, V, W) - (\nabla_W\bar{Q})(V, U, X). \end{aligned} \tag{13}$$

Under second Bianchi identity

$$(\nabla_X\bar{Q})(V, U, W) + (\nabla_V\bar{Q})(X, U, W) + (\nabla_U\bar{Q})(X, V, W) = 0. \tag{14}$$

Using (14) in reduces to

$$curl\ \bar{Q} = -(\nabla_W\bar{Q})(V, U, X).$$

If $\bar{Q}$ is irrotational then $curl\ \bar{Q} = 0$ and we should have

$$(\nabla_W\bar{Q})(V, U, X) = 0$$

$$\implies \nabla_W\{\bar{Q}(V, U)X\} = \bar{Q}(\nabla_W V, U)X + \bar{Q}(V, \nabla_W U)X + \bar{Q}(V, U)\nabla_W X. \tag{15}$$

Put $X = \xi$ in (15) and by virtue of (2), (3) and (12) we have

$$\bar{Q}(V, U)W = A[g(U, W)V - g(V, W)U]. \tag{16}$$

Exercising the inner product of (16) with X

$$\bar{Q}(V, U, W, X) = A[g(U, W)g(V, X) - g(V, W)g(U, X)]. \tag{17}$$

On contraction of the above equation (17) over V and X and using (6) we get

$$S(U, W) = \left[\frac{a(n-1)(\alpha^2 - \rho) - b(n-1)(2\Upsilon + \alpha) - br}{a + b(n-2)}\right] g(U, W). \tag{18}$$

Put $U = W = \xi$ in (18) and using (10), (11) we get the value of Υ as

$$\Upsilon = -(n-1)(\alpha^2 - \rho). \tag{19}$$

We can consequently declare the following:

Theorem 1. *A Ricci soliton (g, ξ, Υ) in irrotational quasi-conformal (LCS) manifold is steady $\alpha^2 - \rho = 0$, shrinking $\alpha^2 - \rho < 0$ and expanding $\alpha^2 - \rho > 0$. i.e., $\alpha^2 + \xi\alpha = 0, \alpha^2 + \xi\alpha > 0, \alpha^2 + \xi\alpha < 0$.*

Theorem 2. *If an (LCS)-manifold is quasi-conformally flat then it is η-Einstein provided $b \neq 0$.*

Proof. Suppose (LCS) is quasi-conformally flat then (6) becomes

$$aR(V,U)W + b[S(U,W)V - S(V,W)U + g(U,W)QV - g(V,W)QU]$$
$$- \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(U,W)V - g(V,W)U] = 0.$$

Put $V = W = \xi$ and using (5), (8) in preceding equation then we get

$$a[\eta(U)\xi + U](\alpha^2 - \rho) + b[(n-1)(\alpha^2 - \rho)\eta(U)\xi + (n-1)(\alpha^2 - \rho)U$$
$$+ \eta(U)Q\xi + QU] - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[\eta(U)\xi + U] = 0.$$

Taking the inner product by X

$$a(\alpha^2 - \rho)[\eta(U)\eta(X) + g(U,X)] + b[(n-1)(\alpha^2 - \rho)\eta(U)\eta(X)$$
$$+ (n-1)(\alpha^2 - \rho)g(U,X) + \eta(U)S(X,\xi) + S(U,X)]$$
$$- \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[\eta(U)\eta(X) + g(U,X)] = 0.$$

$$\Longrightarrow a(\alpha^2 - \rho)[\eta(U)\eta(X) + g(U,X)] + b[(n-1)(\alpha^2 - \rho)\eta(U)\eta(X)$$
$$+ (n-1)(\alpha^2 - \rho)g(U,X) + (n-1)(\alpha^2 - \rho)\eta(Y)\eta(W) + S(U,X)]$$
$$- \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[\eta(U)\eta(X) + g(U,X)] = 0.$$
$$\Longrightarrow bS(U,X) = \left[\frac{r}{n}\left(\frac{a}{n-1} + 2b\right) - (\alpha^2 - \rho)(a + 2b(n-1))\right]\eta(U)\eta(X)$$
$$+ \left[\frac{r}{n}\left(\frac{a}{n-1} + 2b\right) - b(n-1)(\alpha^2 - \rho)\right]g(U,X). \tag{20}$$

$\therefore$ (LCS) manifold is η-Einstein provided $b \neq 0$. $\square$

Let (g, ξ, Υ) be Ricci soliton then

$$(L_\xi g)(U, X) + 2S(U, X) + 2\Upsilon g(U, X) = 0.$$

$$\Longrightarrow \alpha[\eta(U)\eta(X) + g(U, X)] + S(U, X) + \Upsilon g(U, X) = 0.$$

Replacing $U = X = \xi$ in preceding equation we get

$$S(\xi, \xi) + \Upsilon g(\xi, \xi) = 0 \implies \Upsilon = S(\xi, \xi) \implies b\Upsilon = bS(\xi, \xi).$$

Setting $U = X = \xi$ in (20) and equate to above we get

$$\begin{aligned} b\Upsilon = & \left[\frac{r}{n}\left(\frac{a}{n-1} + 2b\right) - (\alpha^2 - \rho)(a + 2b(n-1))\right] \\ & + \left[\frac{r}{n}\left(\frac{a}{n-1} + 2b\right) - b(n-1)(\alpha^2 - \rho)\right](-1) \quad (21) \\ = & -(\alpha^2 - \rho)[a + 2b(n-1) - b(n-1)] \\ = & -(\alpha^2 - \rho)[a + b(n-1)]. \quad (22) \end{aligned}$$

Hence Υ exists if $b = 0$. So we declare the following:

Theorem 3. *The Ricci soliton (g, ξ, Υ) in quasi-conformally flat (LCS)-manifold exists if $b \neq 0$.*

Remark 1. *(i) If $a = 1, b = -\frac{1}{n-2}$ then $\bar{Q}$ decreases to conformal curvature tensor. In this case $\Upsilon = -(\alpha^2 - \rho)$.*
(ii) If $a = 1, b = -\frac{1}{2(n-1)}$ then $\bar{Q}$ decreases to M-projective curvature tensor. In this case $\Upsilon = (n-1)(\alpha^2 - \rho)$.

4. RICCI SOLITONS IN IRROTATIONAL PSEUDO PROJECTIVE (LCS)-MANIFOLDS

Prasad in 2002 [10] defined and studied a pseudo projective curvature tensor field $\bar{P}$ on M of dimension n which includes projective curvature tensor as specific case. It is given by

$$\begin{aligned} \bar{P}(V, U)W = \; & aR(V, U)W + b[S(U, W)V - S(V, W)U] \\ & -\tfrac{r}{n}\left(\tfrac{a}{n-1} + b\right)[g(U, W)V - g(V, W)U]. \quad (23) \end{aligned}$$

Put $W = \xi$ in (23) and using (4), (8) we have

$$\bar{P}(V, U)\xi = \theta[\eta(V)U - \eta(V)U], \quad (24)$$

where $\theta = a(\alpha^2 - \rho) - \Upsilon b - \frac{r}{n}\left(\frac{a}{n-1} + b\right)$.

The rotation (curl) of pseudo projective curvature tensor $\bar{P}$ on a Riemannian manifold is given by

$$\begin{aligned} Rot\ \bar{P} = \quad & (\nabla_X \bar{P})(V,U,W) + (\nabla_V \bar{P})(X,U,W) \\ & +(\nabla_U \bar{P})(X,V,W) - (\nabla_W \bar{P})(V,U,X). \end{aligned} \tag{25}$$

Under second Bianchi identity we get

$$(\nabla_X \bar{P})(V,U,W) + (\nabla_V \bar{P})(X,U,W) + (\nabla_U \bar{P})(X,V,W) = 0, \tag{26}$$

using above in (25), it becomes

$$curl\ \bar{P} = -(\nabla_W \bar{P})(V,U,X).$$

If $\bar{P}$ is irrotational then $curl\ \bar{P} = 0$ and we obtain

$$(\nabla_W \bar{P})(V,U,X) = 0.$$

$$\implies \nabla_W\{\bar{P}(V,U)X\} = \bar{P}(\nabla_W V,U)X + \bar{P}(V,\nabla_W U)X + \bar{P}(V,U)\nabla_W X. \tag{27}$$

Put $X = \xi$ in (27) and by virtue of (2), (3) and (24) we have

$$\bar{P}(V,U)W = \theta[g(U,W)V - (V,W)U]. \tag{28}$$

Taking inner product of (28) with W

$$\bar{P}(V,U,W,X) = \theta[g(U,W)g(V,X) - g(V,W)g(U,X)]. \tag{29}$$

On contraction of equation (29) over V and X, and using (6) we gain

$$S(U,W) = \left[\frac{(a(\alpha^2 - \rho) - b\Upsilon)(n-1)}{a + b(n-1)}\right] g(U,W). \tag{30}$$

Put $U = W = \xi$ in (30) and using (10), (11) we gain the value of Υ

$$\Upsilon = -(n-1)(\alpha^2 - \rho) = -(n-1)(\alpha^2 + \xi\alpha). \tag{31}$$

We can consequently say the following:

Theorem 4. *A Ricci soliton in irrotational pseudo projective (LCS)-manifold is steady, shrinking and expanding accordingly if*

$$\alpha^2 + \xi\alpha = 0, \alpha^2 + \xi\alpha > 0, \alpha^2 + \xi\alpha < 0$$

.

Theorem 5. *A pseudo projectively flat (LCS)-manifold is η-Einstein provided $b \neq 0$.*

Proof. Suppose (LCS) is pseudo projectively flat then (23) can write

$$aR(V,U)W + b[S(U,W)V - S(V,W)U - \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(U,W)V - g(V,W)U] = 0.$$

Put $V = \xi$, using and using (5), (8) in preceding equation then we gain

$$aR(\xi,U)W + b[S(U,W)\xi - S(\xi,W)U - \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(U,W)\xi - g(\xi,W)U] = 0.$$

$$\text{i.e., } a(\alpha^2 - \rho)[g(U,W)\xi - \eta(W)U] + b[S(U,W)\xi - (n-1)(\alpha^2 - \rho)\eta(W)U]$$
$$- \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(U,W)\xi - \eta(W)\eta(U)] = 0.$$

Taking the inner product ξ to precede equation then we obtained

$$a(\alpha^2 - \rho)[-g(U,W) - \eta(W)\eta(U)] + b[-S(U,W) - (n-1)$$
$$\cdot (\alpha^2 - \rho)\eta(W)\eta(U)] + \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(U,W) + \eta(W)\eta(U)] = 0.$$
$$\Longrightarrow bS(U,W) = \left[\frac{r}{n}\left(\frac{a}{n-1} + b\right) - (\alpha^2 - \rho)(a + (n-1)b)\right]\eta(U)\eta(W)$$
$$+ \left[\frac{r}{n}\left(\frac{a}{n-1} + b\right) - a(\alpha^2 - \rho)\right]g(U,W). \tag{32}$$

Thus (LCS)-manifold is η-Einstein. ∎

Next let (ξ, Υ, g) be Ricci soliton then

$$(L_\xi g)(U,W) + 2S(U,W) + 2\Upsilon g(U,W) = 0.$$
$$\Longrightarrow \alpha[\eta(U)\eta(W) + g(U,W)] + S(U,W) + \Upsilon g(U,W) = 0.$$

Put $U = W = \xi$, then the above reduces to

$$S(\xi,\xi) - \Upsilon = 0 \quad \text{i.e.,} \quad \Upsilon = S(\xi,\xi). \tag{33}$$

From (32) and (33), $\Upsilon b = bS(\xi,\xi)$

$$\Upsilon b = \left[\frac{r}{n}\left(\frac{a}{n-1} + b\right) - (\alpha^2 - \rho)(a + (n-1)b)\right]$$
$$+ \left[\frac{r}{n}\left(\frac{a}{n-1} + b\right) - a(\alpha^2 - \rho)\right]$$
$$= -(n-1)b(\alpha^2 - \rho).$$

Suppose $b \neq 0$ then

$$\Upsilon = -(n-1)(\alpha^2 - \rho). \tag{34}$$

Thus we have the following theorem:

Theorem 6. *A Ricci soliton (g, ξ, Υ) in pseudo projectively flat (LCS)-manifold exists if $b \neq 0$ and $\Upsilon = -(n-1)(\alpha^2 - \rho)$.*

Bagewadi et. al. [2], proved the following result:

Theorem 7. *If a Ricci soliton in (LCS)-manifold satisfying $R(\xi, X).\tilde{M}$ then $\Upsilon = -(n-1)(\alpha^2 - \rho)$.*

- *It is shrinking if characteristic vector field ξ is orthogonal to $\nabla\alpha$.*
- *It is shrinking of the angle between characteristic vector field ξ and the gradient vector field $\nabla\alpha$ is acute.*
- *It is shrinking if $\alpha^2 > k \mid \nabla\alpha \mid$, expanding if $\alpha^2 < k \mid \nabla\alpha \mid$, and steady if $\alpha^2 = k \mid \nabla\alpha \mid$.*

The value Υ in (19), (22), (31) and (34) is same as Υ in above theorem. Hence we conclude the following result:

Theorem 8. *If a Ricci soliton in (LCS)-manifold satisfies conditions such as irrotational quasi-conformal, quasi-conformally flat, irrotational pseudo projective and pseudo projective flat then $\Upsilon = -(n-1)(\alpha^2 - \rho)$. Further*

- *It is shrinking if characteristic vector field ξ is orthogonal to $\nabla\alpha$.*
- *It is shrinking of the angle between characteristic vector field ξ and the gradient vector field $\nabla\alpha$ is acute.*
- *It is shrinking if $\alpha^2 > k \mid \nabla\alpha \mid$, expanding if $\alpha^2 < k \mid \nabla\alpha \mid$, and steady if $\alpha^2 = k \mid \nabla\alpha \mid$.*

Shaikh et.al. [5], proved the following result i.e., Ricci solitons using the Eisenhart problem in (LCS)-manifolds.

Theorem 9. *Suppose that in (LCS)-manifold the $(0, 2)$ type tensor field $L_\vartheta g + 2S$ is parallel, where ϑ is a given vector field, then (g, ϑ) yields Ricci soliton and it is given by $\Upsilon = -(n-1)(\alpha^2 - \rho)$.*

Situated on the earlier all results we resolve that the value of $\Upsilon = -(n-1)(\alpha^2 - \rho)$ is same as Theorem (8) and Theorem (9).

5. CONCLUSION

The condition obtained for Ricci solitons of (LCS) manifold all the four methods: semi-symmetry, irrotational, flatness and Eisenhart problem is same i.e. $\Upsilon = -(n-1)(\alpha^2 - \rho)$. Hence the geometry of (LCS) manifold is same i.e. $\Upsilon = -(n-1)(\alpha^2 - \rho)$ in all these cases: semi-symmetry, irrotational, flatness and Eisenhart problem.

References

[1] Amur, K., and Maralabhavi, Y. B. (1977). *On quasi-conformal flat spaces*, Tensor (N.S.) 31, 194.

[2] Ashoka, S. R., Bagewadi, C. S., and Ingalahalli, G. (2014). *A geometry on Ricci solitons in$(LCS)_n$ manifolds*, Differ. Geom. Dyn. Syst. 16, 50-62.

[3] Blaga, A. M. (2018).*Almost η-Ricci solitons in $(LCS)_n$-manifolds*, arXiv:1707.09343, 13, 1-16.

[4] Chandra, S. Hui, S. K., and Shaikh, A. A. (2015). *Second order parallel tensors and Ricci solitons on $(LCS)_n$-manifolds*, Commun. Korean Math. Soc., 30, 123-130.

[5] Hamilton, R. S. (1988). *The Ricci flow on surfaces,* Mathematics and general relativity (Santa Cruz, CA, 1986), Contemp. Math., **71**, American Math. Soc., 237-262.

[6] Hui, S. K., and Chakraborty, D. (2016). *Some types of Ricci solitons on $(LCS)_n$-manifolds*, J. Math. Sci. Advances and Applications, 37, 1-17.

[7] Ingalahalli, G., and Bagewadi, C. S. (2012). *Ricci solitons in α-Sasakian manifolds*, ISRN Geometry, Article ID 421384, 14 pages.

[8] Matsumoto, K. (1989). *On Lorentzian almost paracontact manifolds*, Bull. of Yamagata Univ. Nat. Sci. 12, 151-156.

[9] Mihai, I., and Rosca, R. (1992). *On Lorentzian para-Sasakian manifolds*, Classical Anal., World Sci. Publ., Singapore, 155-169.

[10] Prasad, B. (2002). *A pseudo-projective curvature tensor on a Riemannian manifolds*, Bull. Cal. Math. soc., 94(3), 163-166.

[11] Praveena, M. M., and Bagewadi, C. S. (2016). *A Study on Ricci Solitons in Generalized Complex Space Form* , extracta mathematicae, 31(2), 227–233.

[12] Praveena, M. M., and Bagewadi, C. S. (2016). *On almost pseudo Bochner symmetric generalized complex space forms*, , Acta Math. Acad. Paedagog. Nyhazi. (N.S.) 32, 149–159.

[13] Praveena, M. M., and Bagewadi, C. S. (2017). *On almost pseudo symmetric Kähler manifold*, Palest. J. Math., 44:6(II), 272–278.

[14] Praveena, M. M., Bagewadi, C. S., and Krishnamurthy, M. R. (2021). *Solitons of Kählerian space-time manifolds* , Int. J. Geom. Methods Mod. Phys., 18(2), 81–101.

[15] Shaikh, A. A. (2003). *On Lorentzian almost paracontact manifolds with a structure of the concircular type*, Kyungpook Math. J., 43, 305-314.

[16] Shaikh, A. A., and Baishya, K. K. (2005). *On concircular structure spacetimes*, J. Math. Stat., 1, 129-132.

[17] Yano, K., and Sawaki, S. (1968). *Riemannian manifolds admitting a conformal transformation group*, Journal of Differential Geometry, 2(2), 161-184.

Department of Mathematics, M S Ramaiah Institute of Technology (Affilated to VTU) Bengaluru-560054, Karnataka, INDIA

Department of Mathematics, Jain University, Global Campus-562112, Karnataka, INDIA.

Department of Mathematics, Kuvempu University, Shivamogga, Karnataka, INDIA.